

TOOL: The Bode plot rules for a pair of complex conjugate poles involve approximating

$$|H(j\omega)| = \frac{1}{\left| \left(\frac{j\omega}{\omega_0} \right)^2 + \frac{1}{Q} \left(\frac{j\omega}{\omega_0} \right) + 1 \right|}$$

which has a resonant peak for high Q .

In CTool "[BODE PLOTS:CONJUGATEPOLES:PEAK RESPONSE FREQ DERIVATION](#)", the frequency at the peak is shown to be at a frequency slightly different than ω_0 .

$$\frac{\omega_{\max}}{\omega_0} = \sqrt{1 - \frac{1}{2Q^2}}$$

Note that there is no resonant peak for $Q < 1/\sqrt{2}$.

In CTool "[BODE PLOTS:CONJUGATEPOLES:PEAK RESPONSE MAG DERIVATION](#)", the peak magnitude is shown to be Q scaled by a factor that is small for high Q .

$$|H(j\omega_{\max})| = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}$$

For $Q < 1$, the peak of the resonance is 15% or less and, for $Q < 0.8$, the peak of the resonance is 2% or less. Thus, we may choose to ignore the peak. If not, the above mentioned CTools have tables of peak frequencies and magnitudes that may be used.

For $Q = 1$, $|H_{\max}|$ is 1.15 and $\omega_{\max}/\omega_0 = 1/\sqrt{2} \doteq 0.707$.

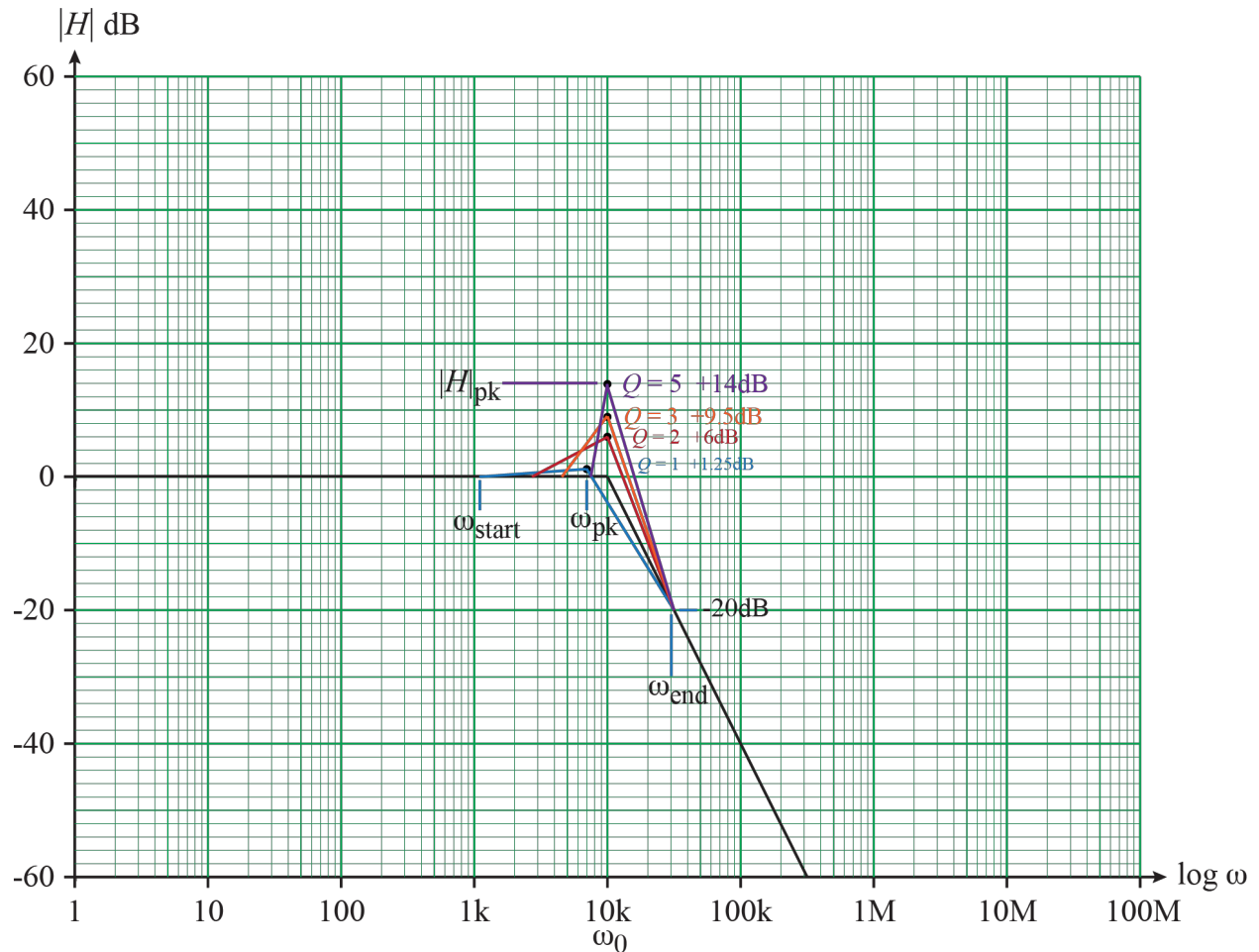
For $Q \geq 2$, $|H_{\max}| \approx Q$ is very accurate and $\omega_{\max}/\omega_0 \approx 1$ with error $< 7\%$ for $Q = 2$.

An approximation for the starting frequency for a line on the Bode plot that ends at the resonant peak gives a frequency in the decade before ω_0 .

$$\frac{\omega_{\text{start}}}{\omega_0} \approx \begin{cases} 0.15Q & 1 \leq Q \leq 6.5 \\ 1 & Q > 6.5 \end{cases}$$

A line from the resonant peak (or the from 0dB at ω_0 for low Q) to a point at -20dB and $\sqrt{10}\omega_0$ is a good approximation for all Q values.

$$\omega_{\text{end}} = 10\omega_0 \text{ and } -20\text{dB}$$



$$Q \geq 2, |H_{\text{max}}| \approx Q \text{ and } \omega_{\text{max}} / \omega_0 \approx 1$$

$$\frac{\omega_{\text{start}}}{\omega_0} \approx \begin{cases} 0.15Q & 1 \leq Q \leq 6.5 \\ 1 & Q > 6.5 \end{cases}$$

$$\omega_{\text{end}} = 10\omega_0 \text{ and } -20\text{dB}$$