TOOL: For a 2-pole low-pass filter with conjugate poles, straight-line approximations work poorly for a Bode plot of magnitude. Thus, we use the actual frequency response curves shown below for the gain as a function of the Q or ζ.

The transfer function for a 2-pole low-pass filter is

$$H(j\omega) = \frac{1}{\left|\left(\frac{j\omega}{\omega_0}\right)^2 + \frac{1}{Q}\left(\frac{j\omega}{\omega_0}\right) + 1\right|}.$$

or, terms of ζ:

$$H(j\omega) = \frac{1}{\left|\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1\right)}.$$

The plot on the next page is for $\omega_0 = 1$. Shift the curve right or left to position it at the desired ω_0. Shift right by (log10) 2π if converting frequency to ω in rad/s.

The peak gain is given exactly by the following formula:

$$|H(j\omega_{\max})| = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}.$$

Note that the peak magnitude is approximately Q for large values of Q.

The frequency of the peak gain is displaced from ω_0 and may be calculated exactly by the following formula:

$$\frac{\omega_{\max}}{\omega_0} = \frac{1}{\sqrt{1 - \frac{1}{2Q^2}}}.$$

Note that the displacement of the maximum frequency is small for high values of Q.

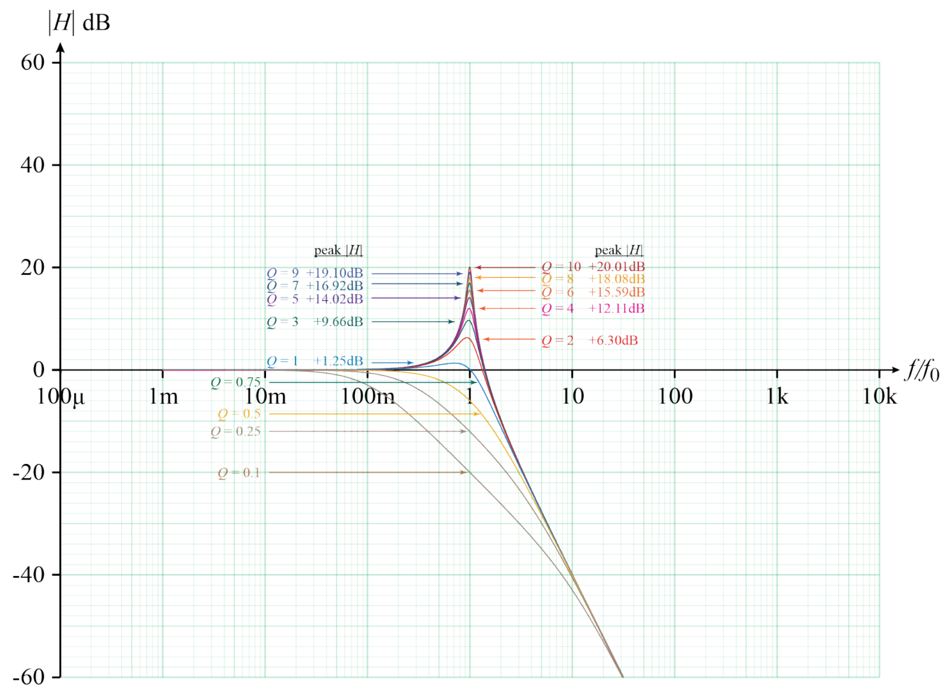

Fig. 1. Conjugate pole magnitude plots versus Q.

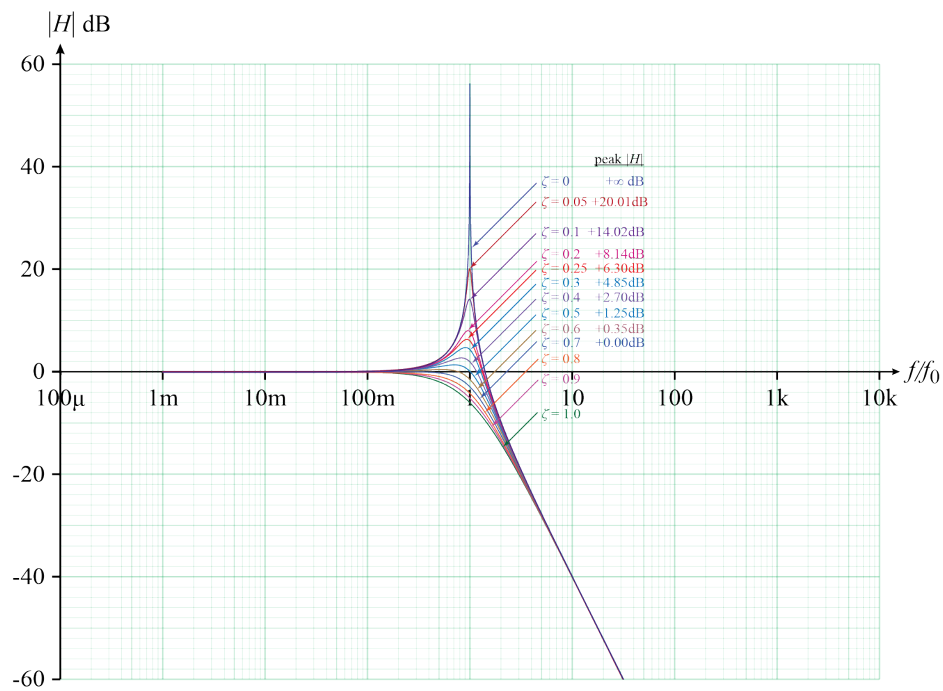

Fig. 2. Conjugate pole magnitude plots versus ζ.