TOOL: The Bode magnitude plot rules for a real zero or pole at ω_0 involve approximating the zero or pole term at low frequency as $\left|\frac{s}{\omega_0}+1\right| = \left|\frac{j\omega}{\omega_0}+1\right| \approx 1$ for $\omega << \omega_0$ and at high frequency as $\left|\frac{s}{\omega_0}+1\right| = \left|\frac{j\omega}{\omega_0}+1\right| \approx \left|\frac{\omega}{\omega_0}\right| = \frac{\omega}{\omega_0}$ for $\omega >> \omega_0$.

The next step is to take the $20\log_{10}$ of the approximation, (i.e., convert to decibels, or dB). Although we are dealing with only one zero or pole term here, taking the logarithm converts the product of zero or pole terms into a sum, since the logarithm of a product is the sum of the logarithms of the multiplicands.

$$20\log_{10}(a \cdot b) = 20\log_{10}(a) + 20\log_{10}(b)$$

This use of the logarithm converts the plotting process into taking the sum of the plots of magnitude or phase

The multiplication by 20 in the dB value is a convenient ploy that yields integer (or half-integer) dB values for integer magnitude values. Table I shows some dB values:

Table I Decibel (or dB) approximate values

value	dB = $20\log_{10}$(value)	approx dB
1	0.00	0
$\sqrt{2}$	3.01	3
2	6.02	6
3	9.54	9.5
4	12.04	12
5	13.98	14
6	15.56	15.5
2π	15.96	16
7	16.90	17
8	18.06	18
9	19.08	19
10	20.00	20

Another remarkable consequence of using a dB scale is that our approximations give curves that always have the same shape. Fig. 1 shows the actual and approximate curves for a single zero at f = 1k Hz or ω = 6.28 r/s.

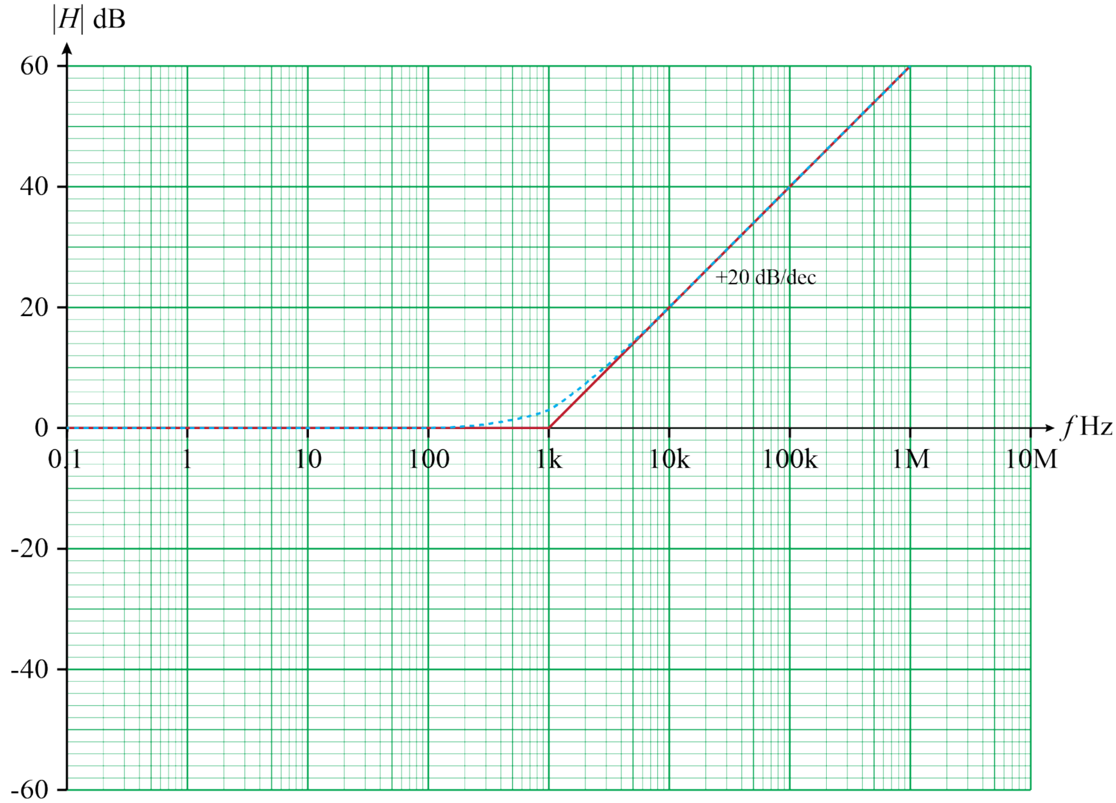

Fig. 1. Bode plot of single real pole (sold lines), and actual curve (dotted line).

The actual curve starts out at 0 dB, gently curves up to a value of 3 dB at $\omega = \omega_0$, and then approach a line that slants upward with a slope of +20 dB per decade (i.e., factor of ten) change in frequency.

The exact curve is well-approximated by the straight lines that meet at $\omega = \omega_0$, as in Fig. 1.

$20\log_{10} 1 = 0$ for $\omega < \omega_0$, a horizontal line at height 0 dB

$20 \cdot (\log_{10}\omega - \log_{10}\omega_0)$ for $\omega > \omega_0$, a line sloping up at 20 dB/decade

The graph below shows the Bode magnitude plot of the following transfer function:

$$|H(s)| = \left|\frac{s+3\text{k}}{(s+1\text{k})(s+20\text{k})}\right| = \frac{3\text{k}}{1\text{k}\cdot 20\text{k}} \cdot \frac{\left|\frac{s}{3\text{k}}+1\right|}{\left|\frac{s}{1\text{k}}+1\right|\left|\frac{s}{20\text{k}}+1\right|}.$$

Using $s = j\omega$ and converting to dB, we have

$$|H(s)|\text{dB} = 20\log_{10}\left(\frac{3}{20}\right) + 20\log_{10}\left(\frac{s}{3\text{k}}+1\right) - 20\log_{10}\left(\frac{s}{1\text{k}}+1\right) - 20\log_{10}\left(\frac{s}{20\text{k}}+1\right).$$

We plot each term and sum the curves (meaning we sum the values at a given frequency ω and then repeat the process for every ω). Fig. 2 shows the resulting plot. The plotting process is straightforward, as the total slope of the curve equals the sum of the slopes of the individual zero and pole curves.

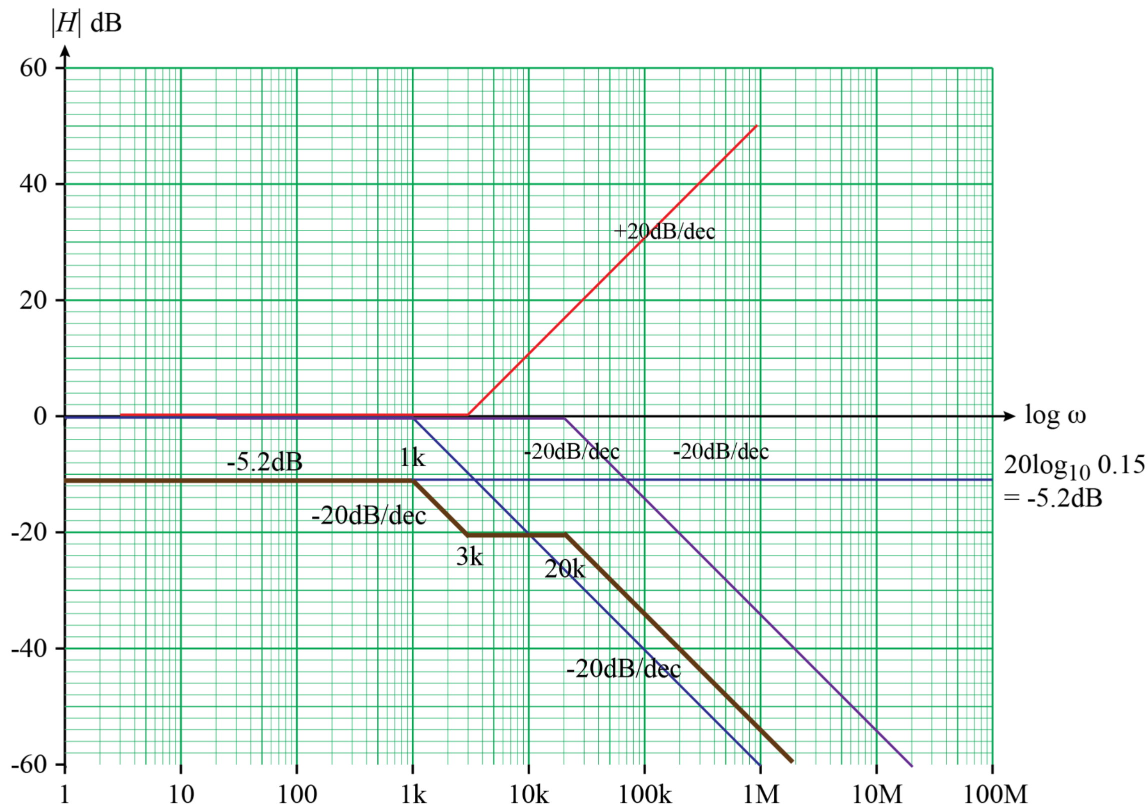

Fig. 2. Bode plot of $H(s)$ with one zero and two poles. Final sum of component curves is shown as heavy brown line.