TOOL: The Bode magnitude plot of a transfer function starts by factoring a transfer function, $H(s)$, into a product of a constant, K, times root terms of form s or $\left(\frac{s}{\omega_0}+1\right)$ in the numerator (zeros) and denominator (poles).

The rules for a real zero or pole at ω_0 involve approximating the zero or pole term:

at low frequency as $\left|\frac{s}{\omega_0}+1\right| = \left|\frac{j\omega}{\omega_0}+1\right| \approx 1$ for $\omega << \omega_0$, and

at high frequency as $\left|\frac{s}{\omega_0}+1\right| = \left|\frac{j\omega}{\omega_0}+1\right| \approx \left|\frac{\omega}{\omega_0}\right| = \frac{\omega}{\omega_0}$ for $\omega >> \omega_0$

The rule for a zero or pole, s, in numerator or denominator requires no approximation:

$|j\omega| = \omega$ for all ω.

The rule for a constant K also requires no approximation:

$|K| = K$ for all ω.

The next step is to take the $20\log_{10}$ of the values from the above rules, (i.e., convert to decibels, dB). Taking the logarithm converts the product of terms into a sum since the logarithm of a product is the sum of the logarithms of the multiplicands.

$$20\log_{10}(a \cdot b) = 20\log_{10}(a) + 20\log_{10}(b)$$

This use of the logarithm converts the plotting process into taking the sum of the plots of individual magnitude terms.

A Bode plot is most readily created for frequency ω in radians/second (r/s), but the Bode plot for frequency f in Hertz is found by simply shifting the plot for ω left by $\log_{10}(2\pi) = 0.798$, which is remarkably close to 0.8 of a decade in frequency. One decade is the width of one bold square on the plot. The logarithmic scale for frequency makes this simple shift possible since it turns division by 2π into subtraction of about 0.8.

$$f = \omega/2\pi \to \log_{10}(f) \approx \log_{10}(\omega) - 0.8$$

The multiplication by 20 in the dB value is a convenient ploy that yields integer (or half-integer) dB values for integer magnitude values. Table I shows some dB values.

Table I Decibel (or dB) approximate values

value	dB = $20\log_{10}$(value)	approx dB
1	0.00	0
$\sqrt{2}$	3.01	3
2	6.02	6
3	9.54	9.5
4	12.04	12
5	13.98	14
6	15.56	15.5
7	16.90	17
8	18.06	18
9	19.08	19
10	20.00	20

Another remarkable consequence of using a logarithmic frequency scale and a dB vertical scale is that our approximations give curves that always have the same shape. Fig. 1 shows the actual (dashed) and approximate (solid red) plots for a single zero, $\left(\frac{s}{\omega_0}+1\right)$ at ω_0 = 1k r/s.

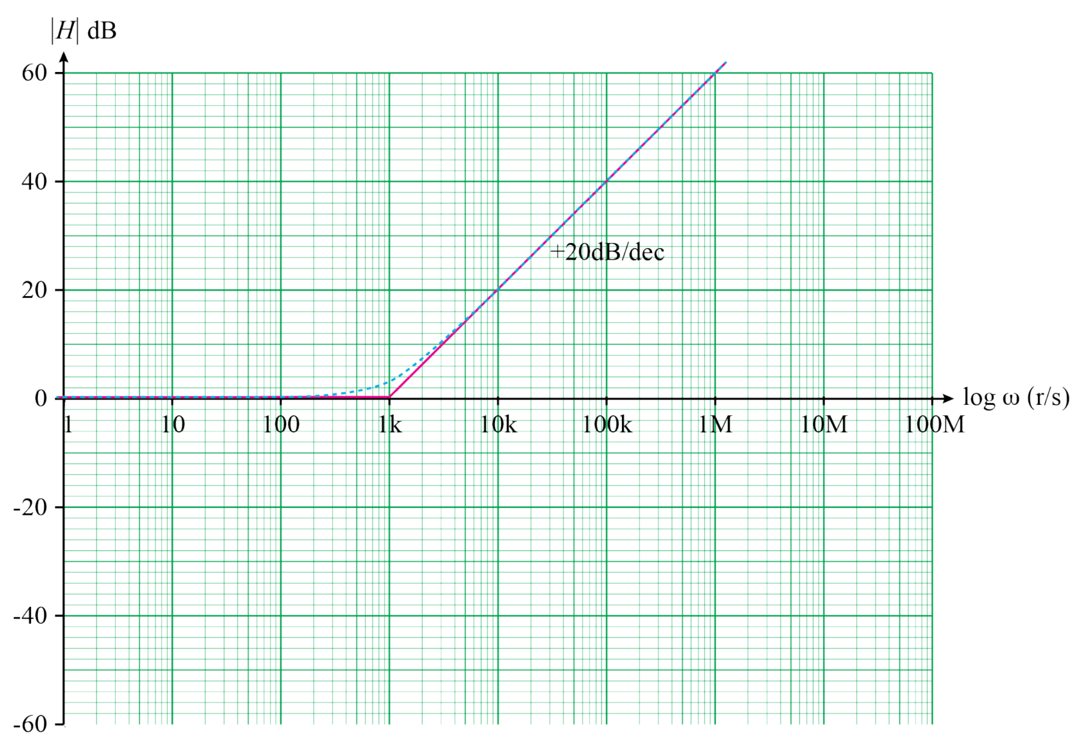

Fig. 1. Bode plot of single real pole (sold line), and actual curve (dotted line).

The actual curve starts out at 0 dB, gently curves up to a value of 3 dB at $\omega = \omega_0$, and then approach a line that slants upward with a slope of +20 dB per decade (i.e., factor of ten) change in frequency.

The exact curve is well-approximated by the straight lines that meet at $\omega = \omega_0$, as shown in Fig. 1.

$20\log_{10} 1 = 0$ for $\omega < \omega_0$, a horizontal line at height 0 dB

$20 \cdot (\log_{10}\omega - \log_{10}\omega_0)$ for $\omega > \omega_0$, a line sloping up 20 dB/decade from $\omega = \omega_0$

To illustrate the entire process, we consider the following example transfer function, $H(s)$:

$$|H(s)| = \left|\frac{1\text{M}\cdot s(s+3\text{k})}{(s+1\text{k})(s+20\text{k})(s+100\text{k})}\right|$$

$$= \frac{1\text{M}\cdot 3\text{k}}{1\text{k}\cdot 20\text{k}\cdot 100\text{k}} \cdot \frac{|s|\left|\dfrac{s}{3\text{k}}+1\right|}{\left|\dfrac{s}{1\text{k}}+1\right|\left|\dfrac{s}{20\text{k}}+1\right|}$$

$$= 0.015 \cdot \frac{|s|\left|\dfrac{s}{3\text{k}}+1\right|}{\left|\dfrac{s}{1\text{k}}+1\right|\left|\dfrac{s}{20\text{k}}+1\right|\left|\dfrac{s}{100\text{k}}+1\right|}$$

Using $s = j\omega$ and converting to dB, we have

$$|H(s)|\text{dB} = 20\log_{10}|0.015| + 20\log_{10}\left|\frac{s}{3\text{k}}+1\right|$$
$$- 20\log_{10}\left|\frac{s}{1\text{k}}+1\right| - 20\log_{10}\left|\frac{s}{20\text{k}}+1\right| - 20\log_{10}\left|\frac{s}{100\text{k}}+1\right|.$$

We plot each term and sum the curves (meaning we sum the values at a given frequency ω and then repeat the process for every ω). Fig. 2 shows the resulting plot.

The plotting process is straightforward, as the total slope of the curve equals the sum of the slopes of the individual zero and pole curves. Starting at a convenient frequency such as \omega = 1 r/s, we calculate the sum of the curves. Moving to the right, we use the sum of the slopes of all the pieces between transitions at zeros or poles. At transitions, the slope changes by some multiple of 20 dB.

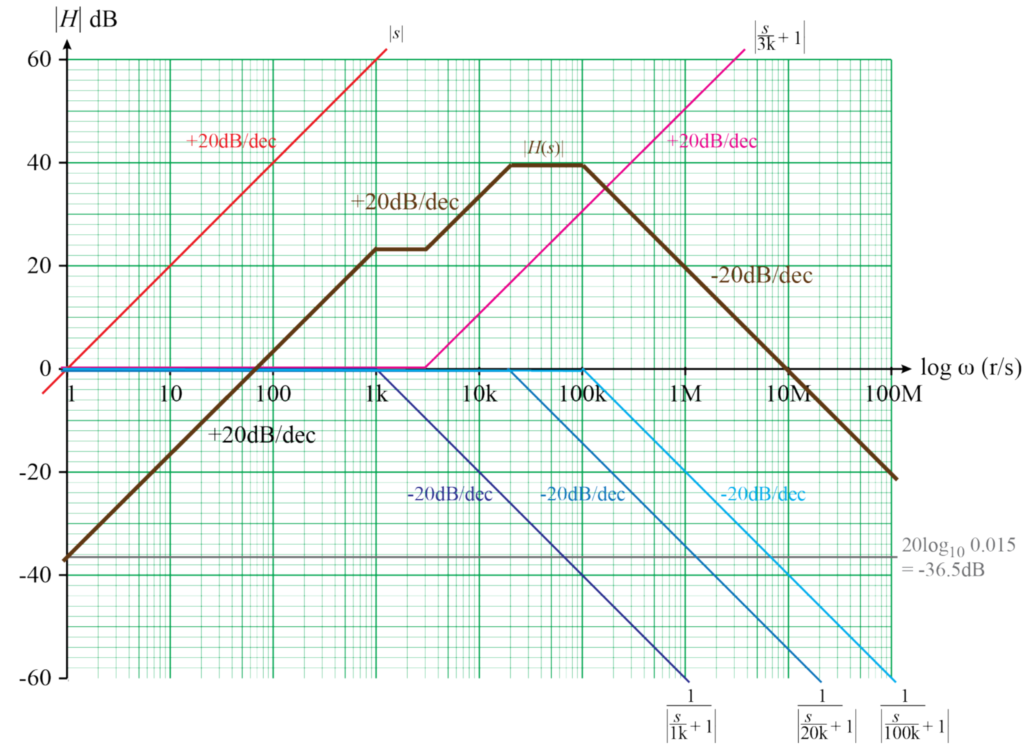

Fig. 2. Bode plot of example $H(s)$ with one zero and two poles. Final sum of component curves is shown as heavy brown line.

Recall that the poles and zeros are all real-valued in this example. The plotting for conjugate poles and zeros is more involved and is treated in a separate Conceptual Tool.