TOOL: The Bode phase plot rules for a transfer function involve approximations for large and small ω similar to those used for magnitude but with different straight-line approximations that are detailed below.

Consider an example transfer function:

$$H(s) = \frac{10s\left(\dfrac{s}{\omega_0}+1\right)\left(\dfrac{s}{\omega_1}+1\right)^2}{\left(\dfrac{s}{\omega_2}+1\right)\left(\dfrac{s}{\omega_3}+1\right)^2} \tag{1}$$

where $\omega_0 = 100\,\mathrm{r/s}$, $\omega_1 = 1\,\mathrm{kr/s}$, $\omega_2 = 10\,\mathrm{kr/s}$, and $\omega_3 = 100\,\mathrm{kr/s}$.

Fig. 1 illustrates the plotting rules for $H(s)$.

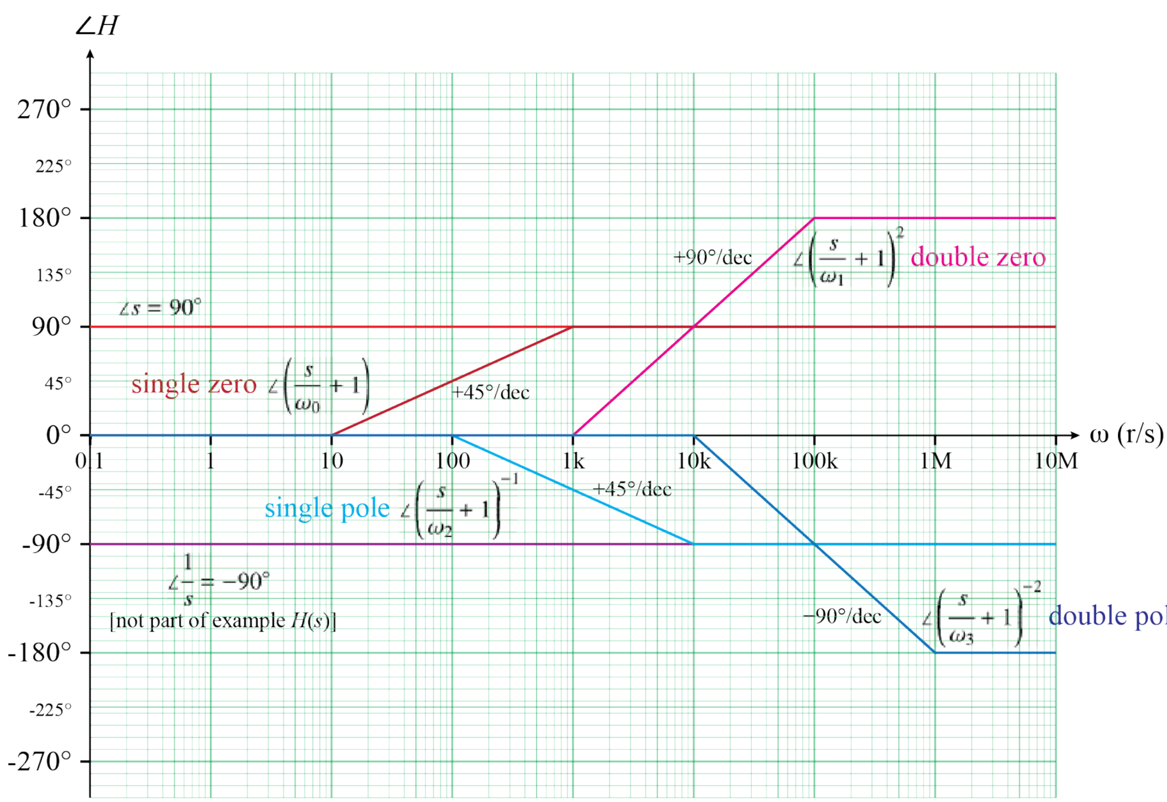

Fig. 1. Examples of Bode phase plot terms.

Rules:

1) ∠constant = 0°. Constant multiplier 10 has angle 0°, so we ignore it.

2) $\angle s = \angle j\omega =$ pure imaginary $= 90°$.

3) $\angle\left(\dfrac{s}{\omega_0}+1\right)$ is centered at ω_0 and extends one decade on either side. Phase angle transitions from 0° to 90°

Fig. 2 shows the sum of the individual phase plot terms as a heavy brown line. To compute the sum, we may start at the left end of the frequency scale and then use the sum of slopes of the individual plots to draw the curve, stopping to change the slope of the sum whenever the slope of an individual curve changes.

Slopes of individual curves are always some multiple of ±45°/dec.

Plots of phase for pole terms are just the negative of the plots of phase for zero terms. So we may just sum all the curves to get the phase for $H(s)$.

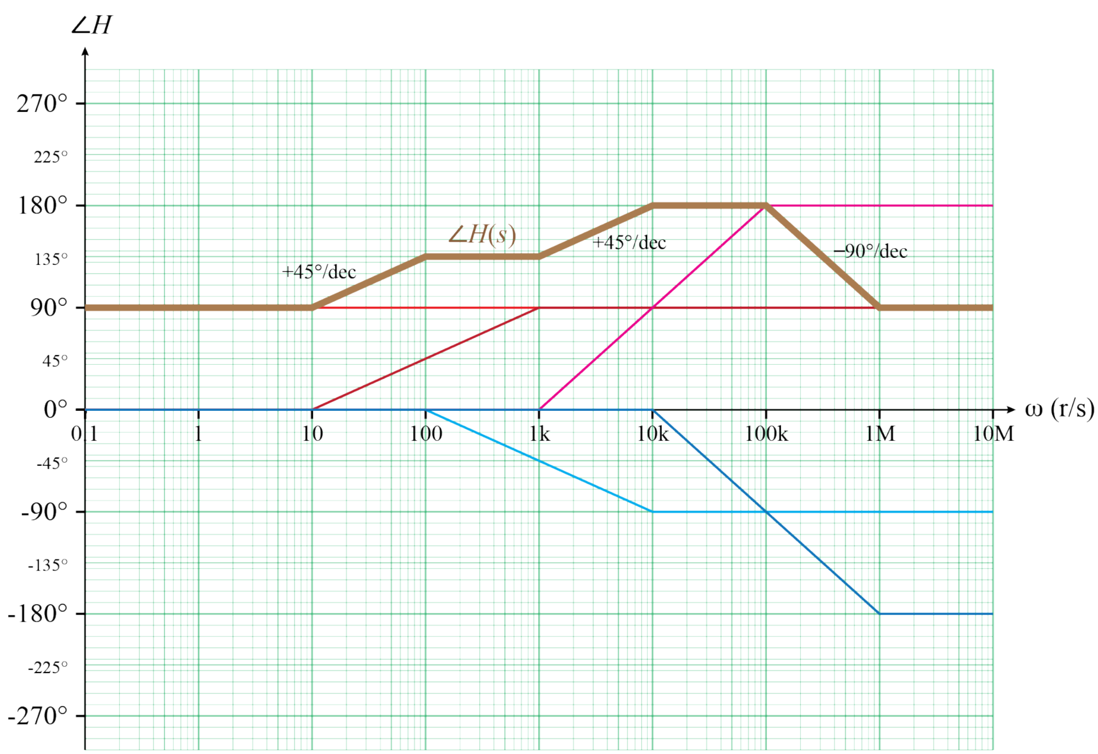

Fig. 2. Summation of Bode phase plot terms yields total $\angle H(s)$, shown in brown.