TOOL: The Bode phase plot rules for a transfer function involve approximations for large and small ω similar to those used for magnitude but with different straight-line approximations that are detailed below.

Consider an example transfer function:

$$H(s) = \frac{10s\left(\frac{s}{\omega_0}+1\right)\left(\frac{s}{\omega_1}+1\right)^2}{\left(\frac{s}{\omega_2}+1\right)\left(\frac{s}{\omega_3}+1\right)^2}$$

where $\omega_0 = 100\,\text{r/s}$, $\omega_1 = 1\,\text{kr/s}$, $\omega_2 = 10\,\text{kr/s}$, and $\omega_3 = 100\,\text{kr/s}$.

Since phase shifts are exponents, as in $Ae^{j\phi}Be^{j\theta} = ABe^{j(\phi+\theta)}$, the phase shift of $H(s)$ is a sum and difference of the multiplied or divided terms of $H(s)$.

$$\angle H(s) = \angle 10 + \angle s + \angle\left(\frac{s}{\omega_0}+1\right) + 2\angle\left(\frac{s}{\omega_1}+1\right) - \angle s - \angle\left(\frac{s}{\omega_2}+1\right) - 2\angle\left(\frac{s}{\omega_3}+1\right)$$

Note that squared terms result in a doubling of the angle. For example,

$$\angle\left(Ae^{j\phi}\right)^2 = 2\phi.$$

Fig. 1 illustrates the following rules for plotting a straight-line approximation of each term in $H(s)$:

1) $\angle 10 = 0°$ since real numbers have an angle of 0° relative to the real axis. Thus, we ignore the real multiplier.

2) $\angle s = \angle j\omega = 90°$ since it lies on the imaginary axis.

3) $\angle\left(\frac{s}{\omega_0}+1\right) \approx \angle 1 = 0°$ for $\omega \le \omega_0/10$,
$\angle\left(\frac{s}{\omega_0}+1\right) \approx 45°/\text{dec}$ rise for $\omega_0/10 \le \omega \le 10\omega_0$, and
$\angle\left(\frac{s}{\omega_0}+1\right) \approx \left(\frac{s}{\omega_0}\right) = \left(\frac{j\omega}{\omega_0}\right) = \angle j = 90°$ for $\omega \ge 10\omega_0$.

The phase curve will be centered at ω_0, unlike the magnitude plot where an elbow occurs at ω_0. Note that we connect the point at the right end of the straight-line approximation for low frequency to the point at the left end of the straight-line approximation for high frequency. This line passes through 45° at ω_0 so we get the exact phase shift for

$$\omega = \omega_0 \text{ where } \angle\left(\frac{s}{\omega_0}+1\right) = \angle(j+1) = 45°.$$

The straight-line approximations may be suboptimal in terms of a least-squares fit of straight lines to the exact phase curve, which is smooth, but starting and ending the phase transition one decade above and below the break frequency of ω_0 is convenient and fairly accurate.

4) A double zero just doubles the angle plot.

5) Plots of phase for pole terms are just the negative of the plots of phase for zero terms. That is, the minus sign is incorporated into the pole plot so we may just sum all the curves to get the phase for $H(s)$.

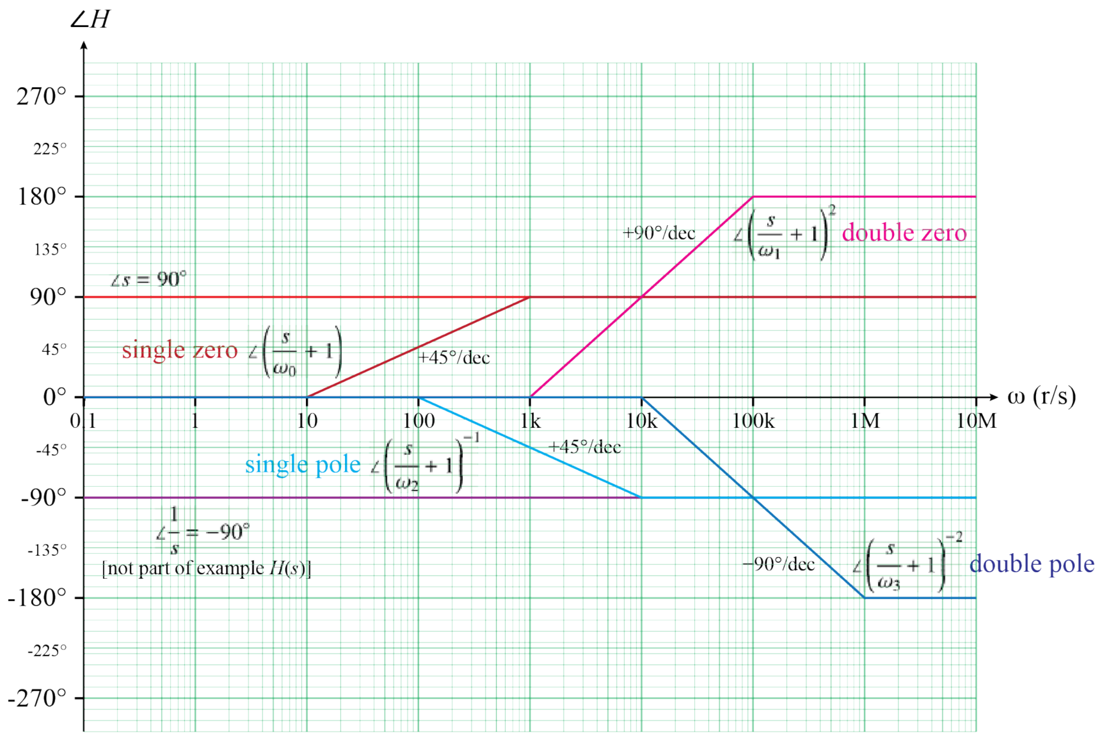

Fig. 1. Examples of Bode phase plot terms.

Fig. 2 shows the sum of the individual phase plot terms as a heavy brown line. To compute the sum, we may start at the left end of the frequency scale and thean use the sum of slopes of the individual plots to draw the curve, stopping to change the slope of the sum whenever the slope of an individual curve changes.

Slopes of individual curves are always some multiple of ±45°/dec.

5) Plots of phase for pole terms are just the negative of the plots of phase for zero terms. That is, the minus sign is incorporated into the pole plot so we may just sum all the curves to get the phase for $H(s)$.

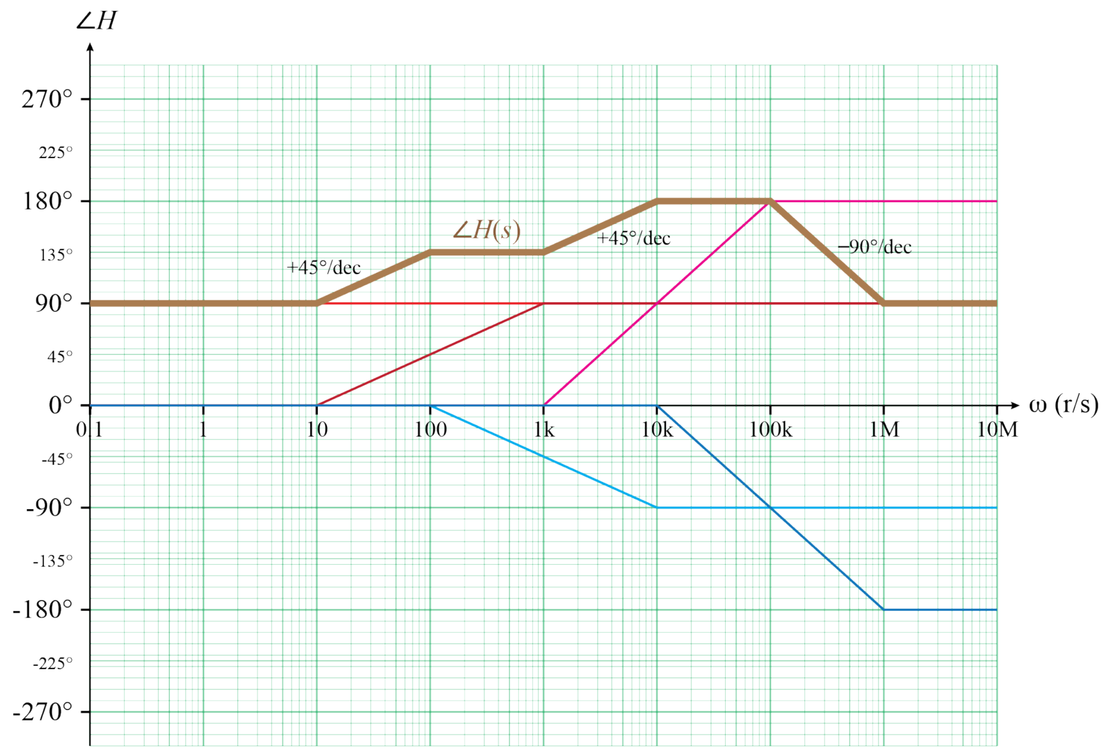

Fig. 2. Summation of Bode phase plot terms yields total $\angle H(s)$, shown in brown.