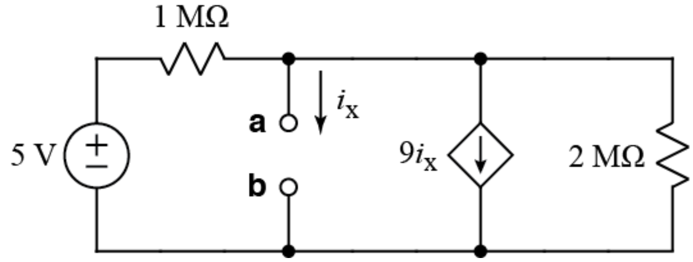
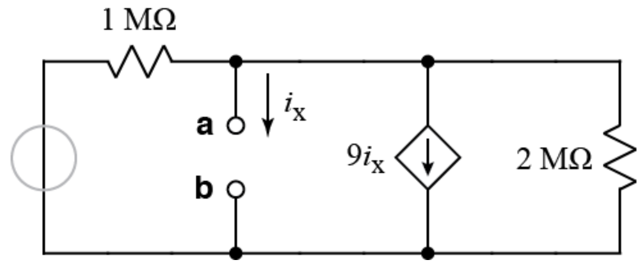
EX: Find the equivalent resistance of the dependent source in the circuit shown below.

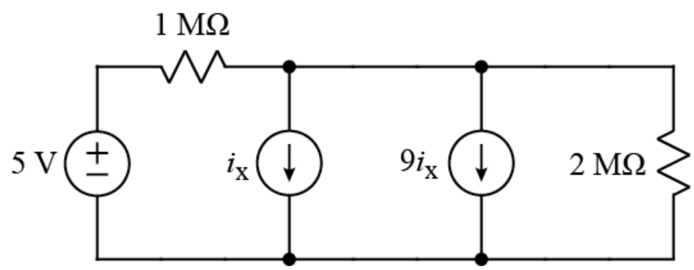


SOL'N: i) Turn off the independent source.

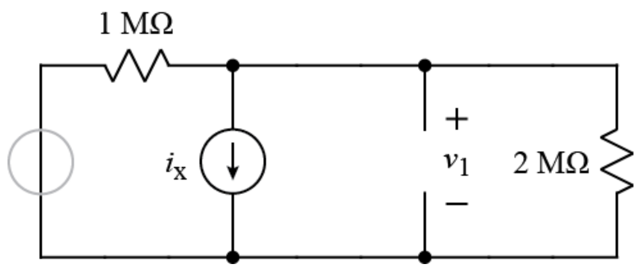


ii) Replace the part of the circuit where dependent variable i_x is measured with an independent current source (since the variable being measured is i_x). Note that the original open circuit from **a** to **b** has become a current source.

iii) Treat the dependent source as an independent current source of value $9i_x$.



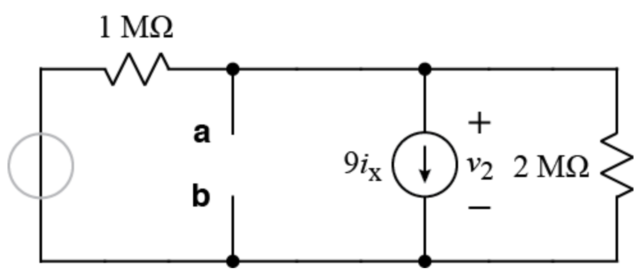
iv) Use superposition to calculate the voltage across the dependent source in terms of i_x. First, turn on the i_x source and turn off the $9i_x$ source. Find the voltage across the dependent source (which is now an open circuit) in terms of i_x. We refer to this voltage as v_1:



The voltage is found by applying Ohm's law.

$$v_1 = -i_x \cdot 1\,\mathrm{M\Omega} \| 2\,\mathrm{M\Omega} = -i_x \cdot 1\,\mathrm{M\Omega} \cdot 1 \| 2 = -i_x \cdot \frac{2}{3}\,\mathrm{M\Omega}$$

v) Second, turn on the αi_x voltage source and turn off the i_x source. (Since the i_x source is a current source, it becomes an open circuit when turned off.) Find the current in the dependent source in terms of i_x. We refer to this current as i_2:

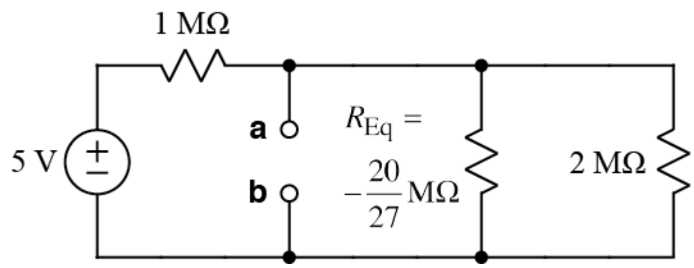


$$v_1 = -9i_x \cdot 1\,\mathrm{M\Omega} \| 2\,\mathrm{M\Omega} = -9i_x \cdot 1\,\mathrm{M\Omega} \cdot 1 \| 2 = -i_x \cdot 6\,\mathrm{M\Omega}$$

vi) Third, sum the two currents above and use Ohm's law to calculate the equivalent resistance of the dependent source as the voltage of the dependent source divided by the current of the dependent source:

$$R_{Eq} = \frac{v_1 + v_2}{i_x} = \frac{-i_x \cdot \frac{20}{27}\,\mathrm{M\Omega}}{i_x} = -\frac{20}{27}\,\mathrm{M\Omega}$$

The equivalent resistance now replaces the dependent source:



We could simplify further by computing the parallel value of the resistors on the right or taking a Norton equivalent of the voltage source and resistor on the left.

NOTE: The negative resistance is used in circuit equations just like a positive resistance. We use the parallel formula, for example, but we keep the minus sign when computing with R_{Eq}.

$$R_{\text{Eq}} \| 2\ \text{M}\Omega = \frac{1\text{M}\Omega}{-\frac{27}{20} + \frac{10}{20}} = \frac{1\text{M}\Omega}{\frac{17}{20}} = \frac{20}{17}\text{M}\Omega$$