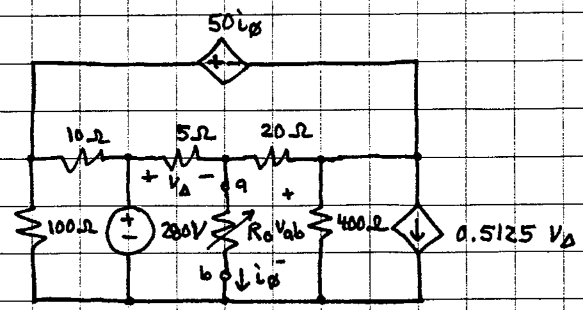
ex:



R_o is adjusted for max power transfer to R_o.

a) Find the value of R_o.

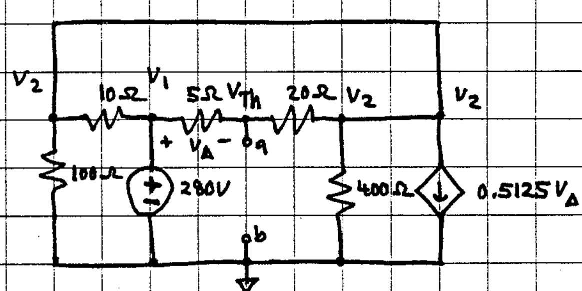
sol'n: Take Thevenin equivalent of circuit without R_o with output a–b terminals as labeled in above diagram. Then set $R_o = R_{Th}$.

Find Thevenin equivalent:

$V_{Th} = v_{ab}$ without R_o (i.e. open circuit)

Then $i_\phi = 0$ and $50 i_\phi = 0V$ is wire.

Now use Node-V method:



v_{Th} not an essential node. Solve for v_1 and v_2. Then use V-divider to find v_{Th}.

$v_1 = 280V$ since we have 280V source. So we really just need to find v_2.

$$\frac{v_2}{100\,\Omega} + \frac{v_2 - 280\,V}{10\,\Omega} + \underbrace{0.5125}_{=\frac{1}{2}+\frac{1}{80}=\frac{41}{80}} v_\Delta + \frac{v_2}{400\,\Omega} + \frac{v_2 - 280\,V}{5\,\Omega + 20\,\Omega} = 0A$$

constraint: $v_\Delta = (280 - v_2) \cdot \frac{5\,\Omega}{5\,\Omega + 20\,\Omega}$ (V-divider) $= \frac{1}{5}$

$$v_2 \left(\frac{1}{100\,\Omega} + \frac{1}{10\,\Omega} - \frac{41}{80}\frac{1}{5\,\Omega} + \frac{1}{400\,\Omega} + \frac{1}{25\,\Omega} \right) = 280V \left(\frac{1}{10\,\Omega} + \frac{1}{25\,\Omega} - \frac{1}{5}\cdot\frac{41}{80\,\Omega} \right)$$

mult by $400\,\Omega$ $\quad v_2 \left(4 + \cancel{40} - \cancel{41} + \cancel{1} + 16 \right) = 280V \left(40 + 16 - 41 \right)$

$$v_2 \; 20 = 280V \cdot 15$$

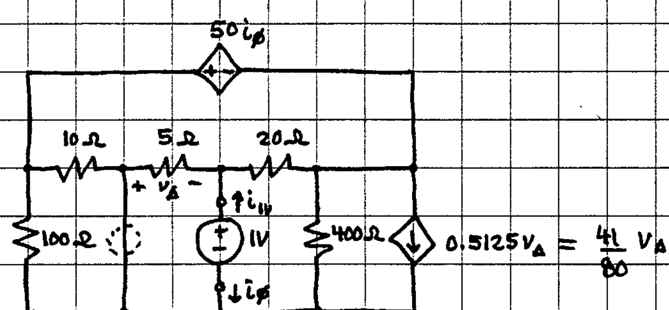
$$v_2 = 14 \cdot 15V = 210\,V$$

$$V_{Th} = 280V - v_\Delta = 280V - \underbrace{(280 - 210)V}_{70V} \cdot \frac{1}{5} = 266\,V$$

Now we find R_{Th}: Connect 1V source at a-b and turn independent 280V source to zero.

$R_{Th} = 1V/(-i_\phi)$ since $i_\phi = -i_{1V}$ where $i_{1V} \equiv i$ into 'a' terminal

Circuit:



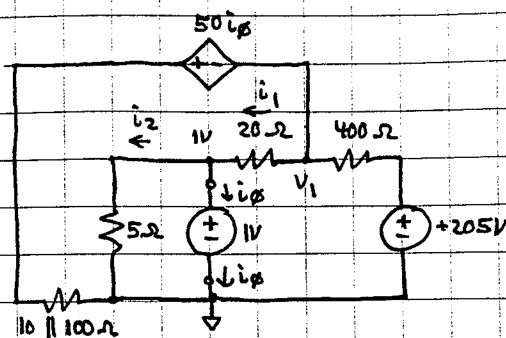
We observe that we have 1V source across $5\,\Omega$ resistor.

$\therefore \; v_\Delta = -1V \qquad \frac{41}{80} v_\Delta = -\frac{41}{80}\,A$

We also have $10\,\Omega \| 100\,\Omega$ on left.

We also can transform $400\,\Omega$ and $0.5125\,v_\Delta$ source to Thevenin equivalent: $R = 400\,\Omega \quad V = -400 \cdot \frac{-41}{80} = +205\,V$

Time to redraw circuit.



Use Node-V method to find v_1:

constraint: $i_1 = i_2 + i_\phi \qquad i_1 = \dfrac{v_1 - 1V}{20\Omega} \qquad i_2 = \dfrac{1V}{5\Omega}$

$$\therefore\ i_\phi = i_1 - i_2 = \frac{v_1 - 1V}{20\Omega} - \frac{1V}{5\Omega} = \frac{v_1 - 5V}{20\Omega}$$

Node v_1: $\dfrac{v_1 - 1V}{20\Omega} + \dfrac{v_1 - +205V}{400\Omega} + \dfrac{v_1 + 50 i_\phi}{10 \| 100\Omega} = 0\,A$

$$v_1\left(\frac{1}{20\Omega} + \frac{1}{400\Omega} + \frac{1}{10\|100\Omega} + \frac{50/20}{10\|100\Omega}\right) = \frac{1V}{20\Omega} + \frac{205V}{400\Omega} + \frac{50\cdot 5\,V}{20\cdot 10\|100\Omega}$$

mult by 400 $\quad v_1\left(20 + 1 + \frac{\cancel{400}\cdot 110}{\cancel{1000}} + \frac{\cancel{50}}{\cancel{20}}\cdot\frac{110\cdot\cancel{400}}{\cancel{1000}\cdot\cancel{50}}\right) = 20 + 205 + \frac{\cancel{250}\cdot 110\cdot\cancel{400}^{100=5}}{\cancel{20}\cdot\cancel{1000}_{=\cancel{4}}}\,V$

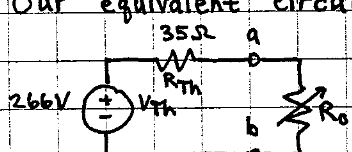
$$v_1(20 + 1 + 44 + 110) = 20 + 205 + 550\ V$$

$$v_1\ 175 = 775V$$

$$i_{1V} = -i_\phi = \frac{v_1 - 5V}{20\Omega} = \frac{\frac{775V}{175} - 5V}{20\Omega}$$

$$R_{Th} = \frac{1V}{i_{1V}} = \frac{-20\Omega}{\frac{775}{175} - 5} = 35\Omega$$

Finally: Our equivalent circuit:



Max power to R_o when $\boxed{R_o = R_{Th} = 35\Omega}$

b) When $R_o = R_{Th}$ the power delivered to R_o (i.e. the max power asked for, for part (b)) $= \dfrac{V_{Th}}{2R_{Th}} \cdot \dfrac{V_{Th}}{2} = i \cdot v$

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}} \quad \text{(this formula always holds)}$$

$$= \frac{266^2}{4 \cdot 35} W = 505.4\,W$$

c) Find power delivered by 280V source when R_o adjusted for max power, (i.e. $R_o = R_{Th} = 35\Omega$, as above).

soln: We know $v_{ab} = \dfrac{V_{Th}}{2} = 133V$.

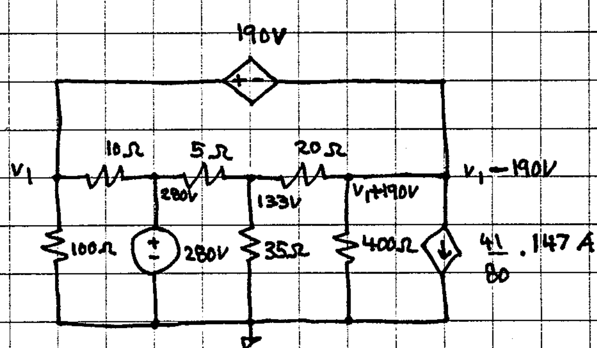
We also know $i_\phi = \dfrac{v_{ab}}{R_o} = \dfrac{\frac{V_{Th}}{2}}{R_o} = \dfrac{133V}{35\Omega}$

And $\therefore$ $v_\Delta = 280V - v_{ab} = 280 - 133\,V = 147V$

$$0.5125\,v_\Delta = \frac{41}{80} v_\Delta = \frac{41}{80} \cdot 147V$$

$$50 i_\phi = 50 \cdot \frac{133V}{35\Omega} = \frac{10}{7} \cdot 133 = 10 \cdot \frac{19 \cdot 7}{7} = 190V$$

Circuit:



Use Node-V to find v_1:

$$\frac{v_1}{100\Omega} + \frac{v_1 - 280V}{10\Omega} + \frac{41}{80} \cdot 147 + \frac{v_1 - 190V}{400\Omega} + \frac{(v_1 - 190V) - 133V}{20\Omega} = 0A$$

$$v_1\left(\frac{1}{100\Omega} + \frac{1}{10\Omega} + \frac{1}{400\Omega} + \frac{1}{20\Omega}\right) = \frac{280V}{10\Omega} - \frac{41 \cdot 147}{80} + \frac{190V}{400\Omega} + \frac{323V}{20\Omega}$$

mult by 400: $v_1 \left(4 + 40 + 1 + 20 \right) = 280 \cdot 40 - 41 \cdot 147 \cdot 5 + 190 - 20 \cdot 323$

$$v_1 \, 65 = 11200 - 30135 + 190 + 6460$$

$$v_1 = -\frac{12285}{65} = -189 V$$

The current flowing out of the 280V source is:

$$i = \frac{280V - v_1}{10\Omega} + \frac{280 - v_{ab}}{5\Omega} = \frac{280 - -189}{10} + \frac{280 - 133}{5}$$

$$i = \frac{469}{10} + \frac{147}{5} = \frac{763}{10} = 76.3 A$$

$$p = i \cdot v = 76.3A \cdot 280V = 21.364 \text{ kW}$$