EX: Find $\text{Re}[\frac{6+j3}{2-j4}e^{jx}]$, (i.e., find the real part) where "x" is real

ANS: 1.5 cos(x + π/2)

SOL'N: We may take one of several different approaches to convert the quantity inside the brackets into the form $a + jb$ (where a is our final answer). We'll take the approach of rationalizing the fraction.

$$\text{Re}[\frac{6+j3}{2-j4}e^{jx}] = \text{Re}[\frac{6+j3}{2-j4}\frac{2+j4}{2+j4}e^{jx}]$$

$$= \text{Re}[\frac{12-12+j(24+6)}{2^2+4^2}e^{jx}]$$

$$= \text{Re}[\frac{j30}{20}e^{jx}]$$

We now use Euler's formula to expand the complex exponential:

$$= \text{Re}[\frac{j30}{20}\{\cos(x)+j\sin(x)\}]$$

$$= \text{Re}[-1.5\sin(x)+j1.5\cos(x)]$$

Our final answer is the real part, which we may express in several ways.

$$\text{Re}[\frac{6+j3}{2-j4}e^{jx}] = -1.5\sin(x) \text{ or}$$

$$\text{Re}[\frac{6+j3}{2-j4}e^{jx}] = 1.5\cos(x+\pi/2) = 1.5\cos(x+90°)$$

NOTE: A curious feature of this problem is that the fraction consisting of complex numbers is purely imaginary. We now examine this symbolically.

$$k\cdot\frac{a+jb}{b-ja} = k\cdot\frac{j(b-ja)}{b-ja} = jk$$

Whenever the numerator and denominator of a fraction have the above pattern, we will find that the result is purely imaginary. Note the necessary minus sign.