TOOL: The imaginary number j (or for mathematicians, i) has the following 10 uses.

TOOL 9: $j = \sqrt{-1}$ allows us to take the square root of negative numbers.

$$j = \sqrt{-1} \qquad j^2 = -1 \qquad \frac{1}{j} = -j$$

TOOL 8: j allows us to write values of all roots of polynomials as complex numbers.

polynomial roots all of form $a + jb$

TOOL 7: j creates a second "imaginary" axis perpendicular to the real axis, meaning complex numbers are vectors.

j = token indicating vertical part of complex number vector

TOOL 6: j defines a rule for multiplying 2-dimensional vectors to obtain a 2-dimensional vector, (unlike dot product = scalar, and cross product = 3-dimensional vector).

$$(a + jb)(c + jd) = ac - bd + j(ad + bc)$$

TOOL 5: j placed in front of a complex number (i.e., multiplying by j), rotates the complex number vector by 90°.

$$j(a + jb) = -b + ja \qquad (a, b)\circ(-b, a) = 0$$

TOOL 4: j allows us to use complex exponentials to write trigonometric functions.

$$e^{j\phi} = \cos\phi + j\sin\phi \qquad \cos\phi = \frac{e^{j\phi} + e^{-j\phi}}{2}$$

TOOL 3: j allows us to shift Real $\cos\phi$ and Imaginary $\sin\phi$ to Real $-\sin\phi$ and Imaginary $\cos\phi$, a phase shift of 90°.

$$je^{j\phi} = j(\cos\phi + j\sin\phi) = -\sin\phi + j\cos\phi = \cos(\phi + 90°) + j\sin(\phi + 90°)$$

TOOL 2: j allows us to capture the magnitude and phase of a sinusoid as a complex exponential called a "phasor".

$$P[A\cos(\omega t + \phi)] = Ae^{j\phi}$$

TOOL 1: j in phasors allows us to sum trigonometric functions without identities.

$$A\cos(\omega t + \phi) + B\sin(\omega t + \theta) = P^{-1}[Ae^{j\phi} - jBe^{j\theta}]$$

TOOL 0: j in phasors allows us to write derivatives as multiplication by $j\omega$.

$$P[A\cos(\omega t + \phi)] = Ae^{j\phi} \qquad P\left[\frac{d}{dt}A\cos(\omega t + \phi)\right] = j\omega \cdot Ae^{j\phi}$$