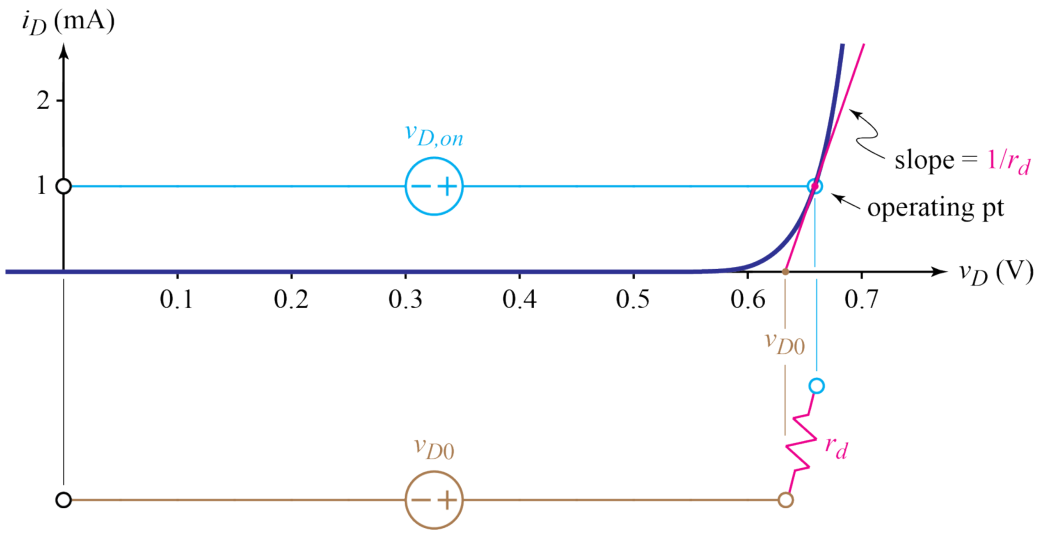
TOOL: Diode and transistor models are based on three tools: linearization, Thévenin equivalent models, and superposition.

The idea of linearization is to use nonlinear devices either with small input signals or in conditions where the *i*-*v* curve is nearly a straight line. If the *i*-*v* curve is nearly a straight line, we may model it approximately with a Thévenin equivalent. Finally, if we use superposition of DC and AC responses, the DC model can be a voltage source (that includes the effect of the Thévenin resistance), and the AC model is the Thévenin resistance.

The figure below shows an *i*-*v* curve for a diode and the linear modeling components of its Thévenin equivalent, $V_{D,on} = 0.7$ V and $r_d = v_T/I_D$ with $v_T \approx 26$ mV.



The Thévenin equivalent depends on the operating point, and finding the operating point involves some approximations. For a diode we may assume a DC voltage drop, $v_{D,on}$, across the diode equal to, say, 0.7V for moderate current or higher power supply current. If the other voltages in the circuit are distinctly higher than the error in $v_{D,on}$, then the error in our Thévenin equivalent values will be small. In the above figure, the correct $v_{D,on}$ is 0.66 V, so the error in the value of $v_{D,on}$ is only 0.04 V.

Because we are usually most interested in the AC behavior of the circuit (i.e., the response to input signals), and the AC behavior depends on the dynamic resistance, we get fairly accurate results by assuming a fixed 0.7 V value for the diode's *v*-drop.

Although we could use a complete Thévenin equivalent model for a diode (or any nonlinear device), it is simpler when using superposition to use just a voltage source, $v_{D,on}$, for DC conditions, and just a resistance for AC conditions (when the DC source will be turned off). Note that, because the current, $i_{D,on}$, in the Thévenin equivalent model is nonzero, the DC conditions voltage source, $v_{D,on}$, is *not* the same as the Thévenin equivalent voltage source, v_{D0}. There is a voltage drop across the Thévenin equivalent resistance caused by $i_{D,on}$. Fortunately, we rarely need to know the value of v_{D0}.

The Thévenin equivalent or dynamic resistance at an operating point on an *i*-*v* curve is the inverse of the tangent line at the operating point. From calculus, the slope of the tangent line is the derivative of the curve.

For a p-n junction, the *i*-*v* curve is an exponential function:

$$I_D = I_S(e^{v_D/v_T} - 1) \approx I_S(e^{v_D/v_T})$$

where

$I_S \equiv$ reverse saturation current for diode

$v_T \equiv$ thermal voltage $\equiv kT/q$ = 25.850285 mV at 300 °K or 80.33 °F

$k \equiv$ Boltzmann constant = $13.80558 \cdot 10^{-24}$ J/°K [1]

$T \equiv$ temperature in °K

$q \equiv$ electron charge = $160.217733 \cdot 10^{-21}$ C

It would appear that we will need to know I_S as well as v_D in order to derive a linear model, but the math simplifies conveniently. The dynamic resistance is the inverse of the dynamic conductance:

$$g_d = \frac{dI_D}{dv_D} = I_S \frac{1}{v_T} e^{v_D/v_T} \approx \frac{I_D}{v_T}$$

Taking the inverse gives the small signal resistance.

$$r_d \approx \frac{v_T}{I_D}$$

At room temperature, we have

$$r_d \approx \frac{26 \text{ mV}}{I_D}.$$

If the diode should get hot enough to boil water (around 100 °C), the value of v_T and, therefore, of r_d will be reduced to about 5/7 of its room temperature value.

For our example in the figure above, we calculate the small-signal resistance as

$$r_d \approx \frac{26 \text{ mV}}{1\text{mA}} = 26\ \Omega.$$

To verify that this is consistent with the figure, we may calculate the slope of the tangent line (in magenta) from the operating point and the intersection of the tangent line with the v-axis.

$$g_d \approx \frac{\Delta I_D}{\Delta v_D} \approx \frac{1 \text{ mA} - 0 \text{ mA}}{0.66 \text{ V} - 0.63 \text{ V}} = \frac{1}{30}\ \Omega^{-1}$$

or

$$r_d \approx \frac{1}{30} = 30\ \Omega^{-1}$$

This is consistent after rounding. The calculation using i_D and v_T is more accurate in general, however.

Having found the model, we use it as follows:

DC model is voltage source $v_{D,on}$ alone, typically taken to be 0.7 V

AC model is resistance r_d alone, where $r_d = \dfrac{26 \text{ mV}}{I_D}$

DC+AC model (rarely used) is Thévenin equivalent $v_{D0} = v_{D,on} - r_d \cdot I_D$ and r_d

REF: [1] H. G. Jerrard and D. B. McNeill, *Dictionary of Scientific Units*, 6th Ed., London, UK: Chapman & Hall, 1992.