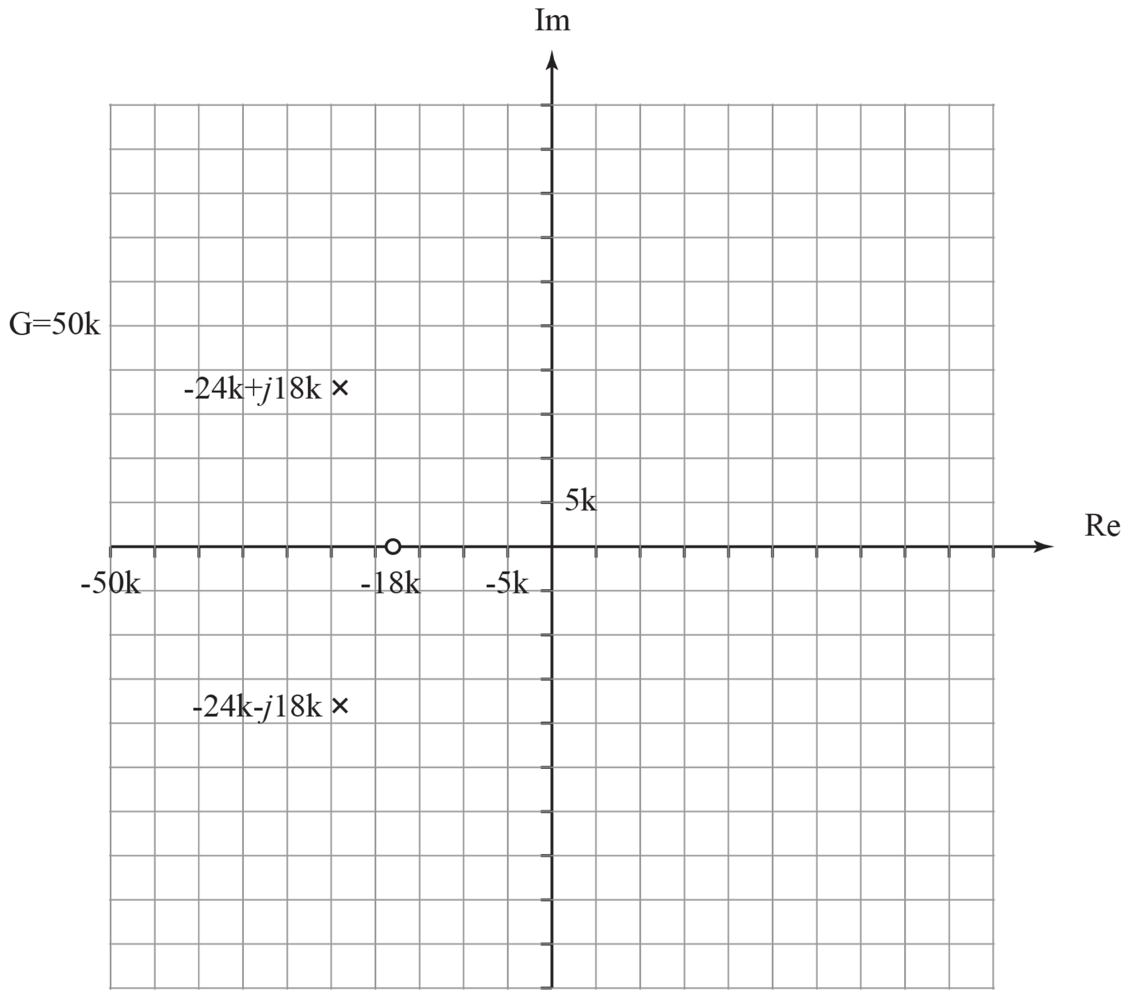
EX: Find the magnitude response plot and phase response plot for the following filter using distances and angles from poles and zeros to a point at $j\omega$ on the imaginary axis.

$$H(j\omega) = 50\text{k}\frac{s+18\text{k}}{(s+24\text{k}+j18\text{k})(s+24\text{k}-j18\text{k})}$$

SOL'N: The pole-zero diagram for the filter:



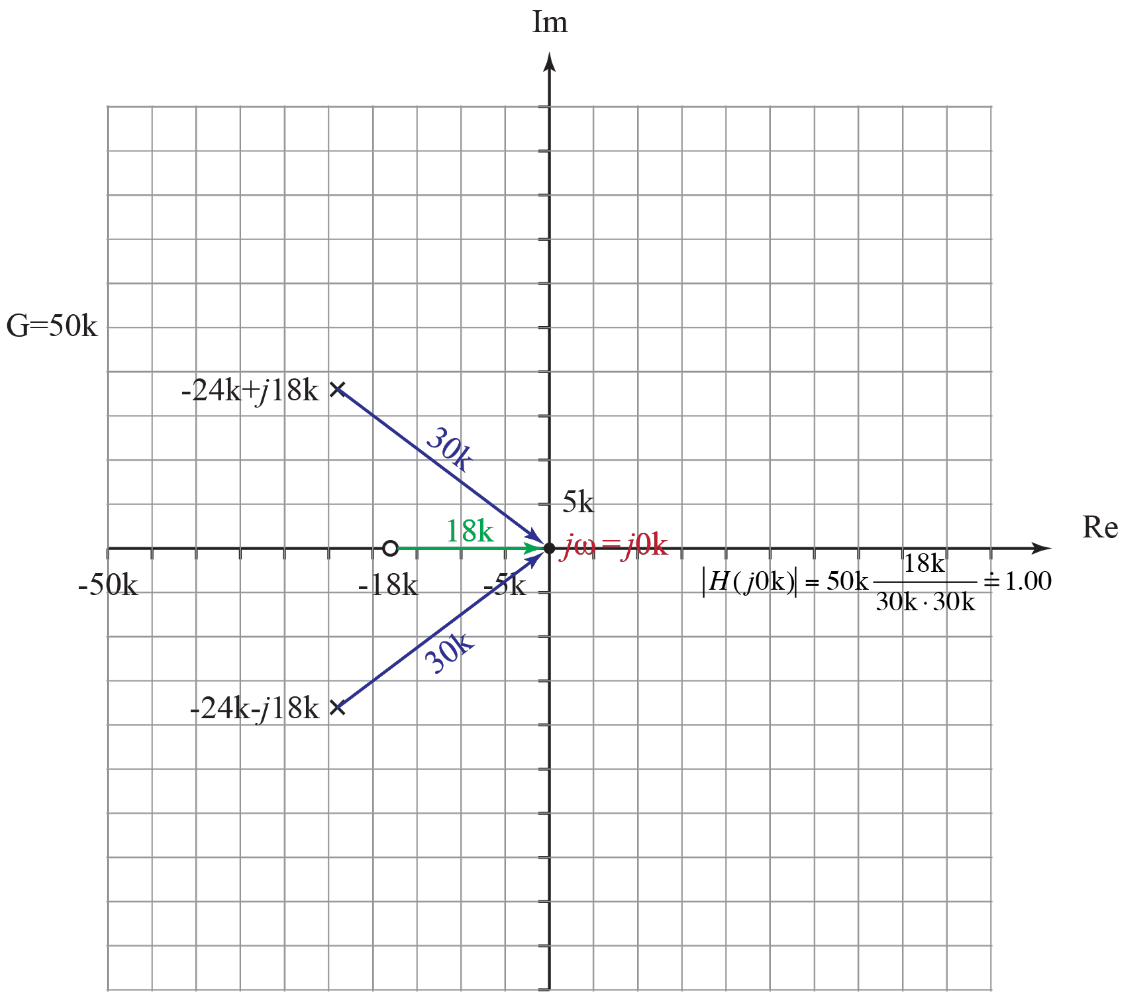
For the magnitude, we can take magnitudes of each term.

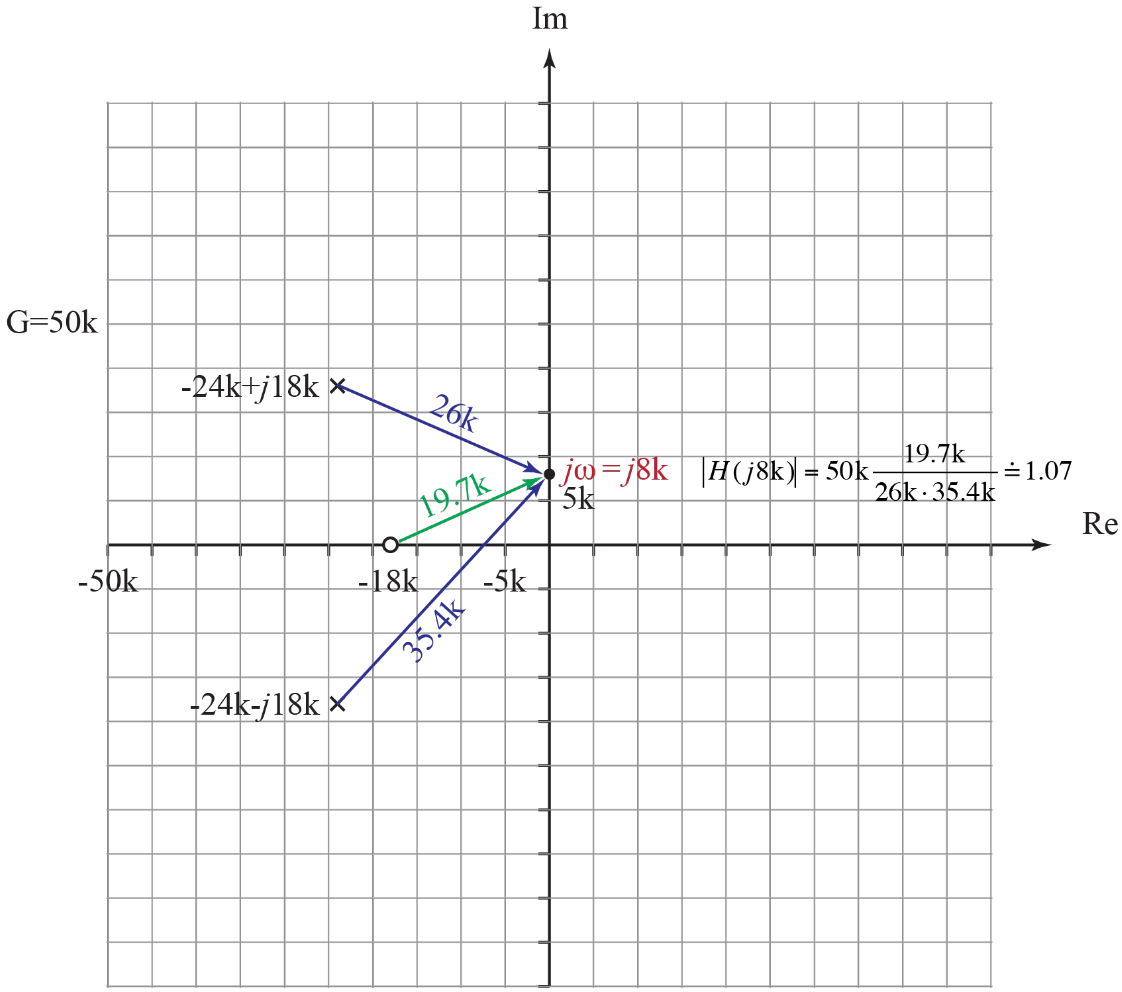
$$|H(j\omega)| = 50\text{k}\frac{|s+18\text{k}|}{|s+24\text{k}+j18\text{k}||s+24\text{k}-j18\text{k}|} \quad s = j\omega$$

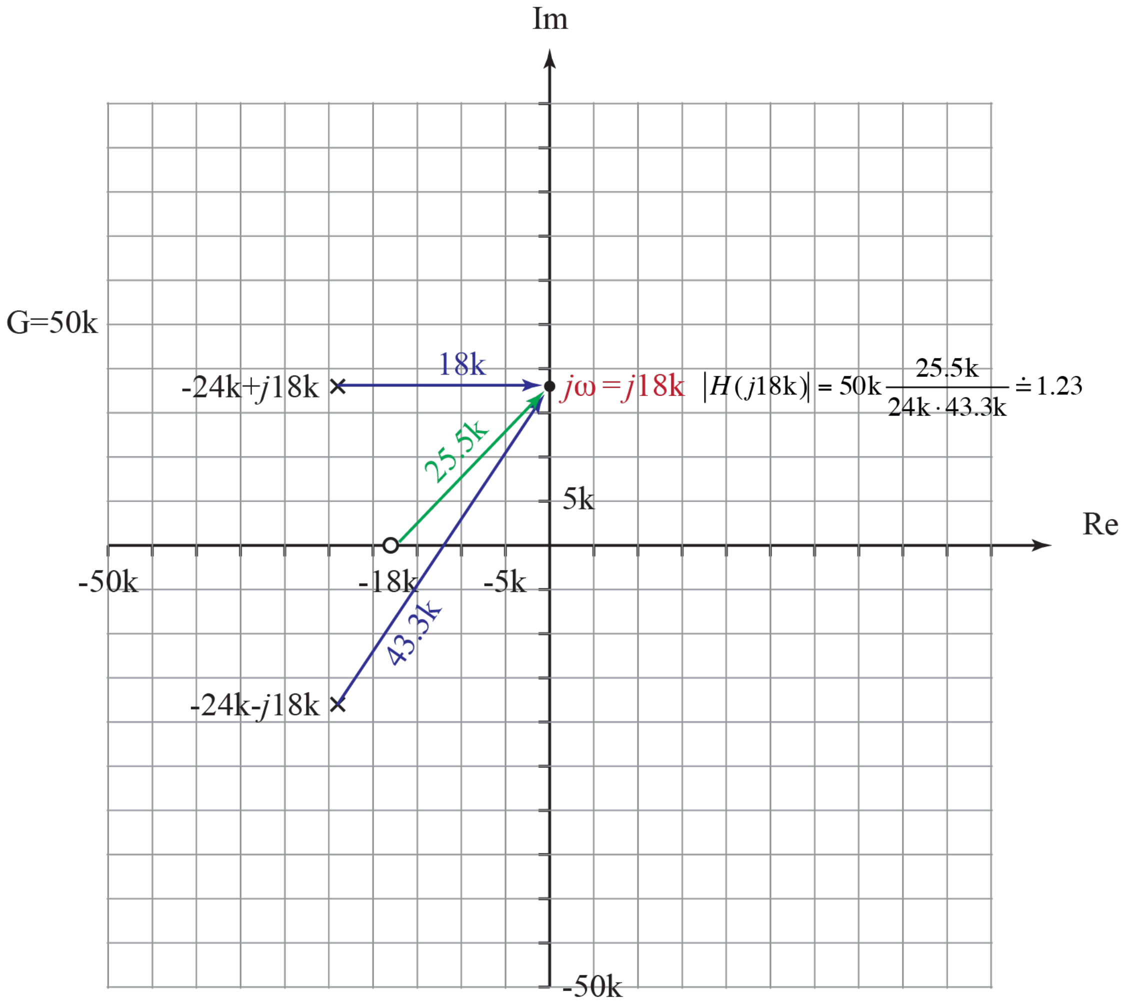
Writing the pole and zero terms as subtractions reveals that the magnitudes correspond to the distances from $j\omega$ to poles and zeros.

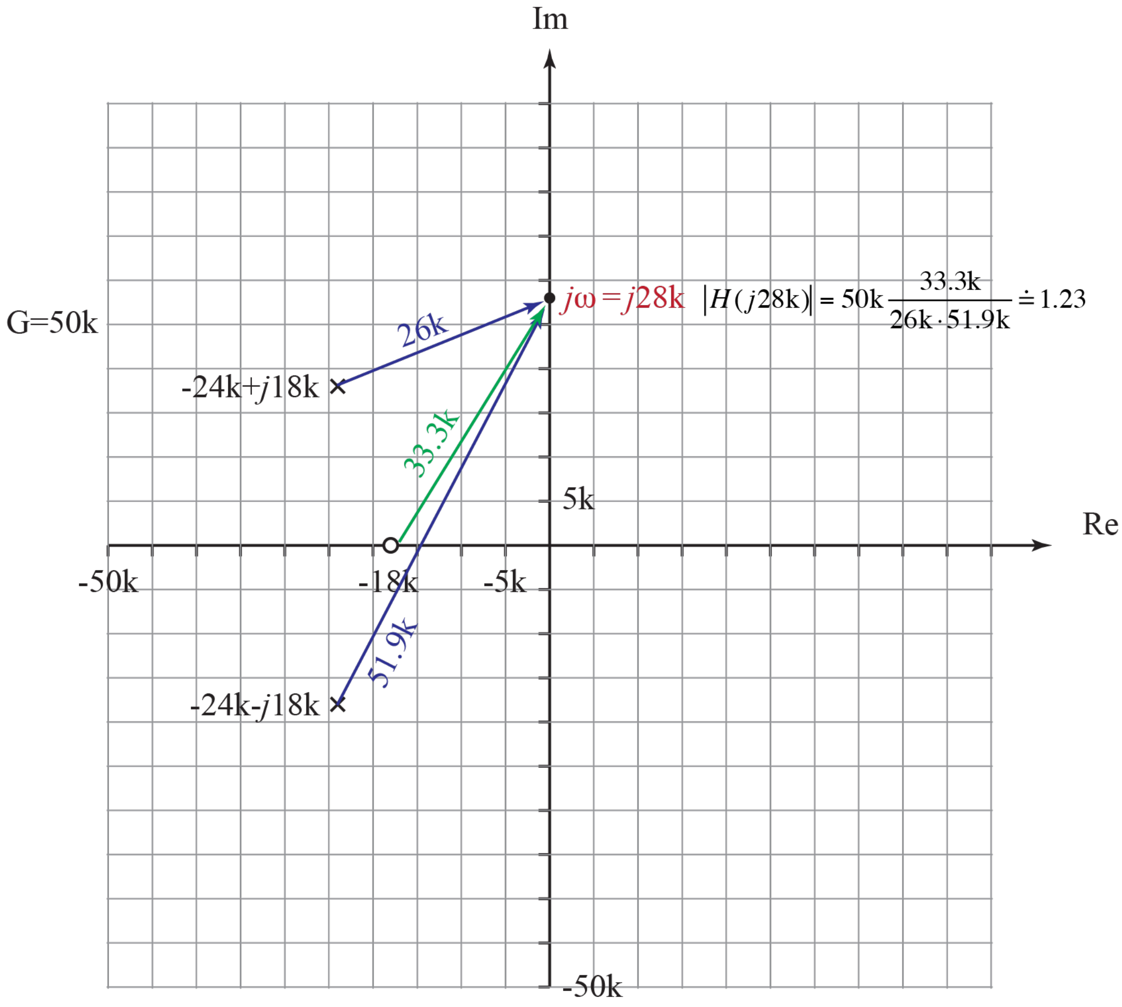
$$|H(j\omega)| = 50\text{k}\frac{|s - -18\text{k}|}{|s - -24\text{k} - j18\text{k}||s - -24\text{k} + j18\text{k}|} \quad s = j\omega$$

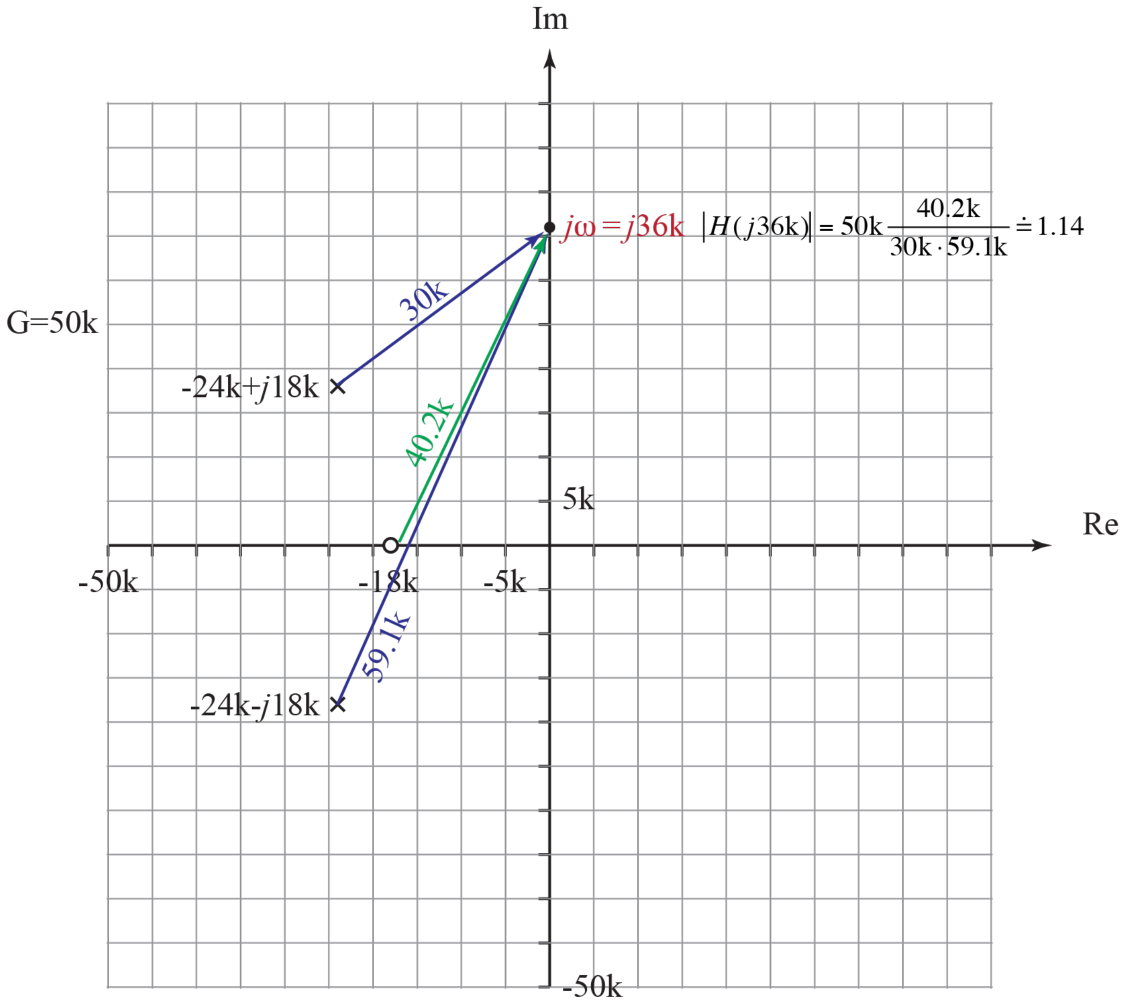
The magnitude calculations:

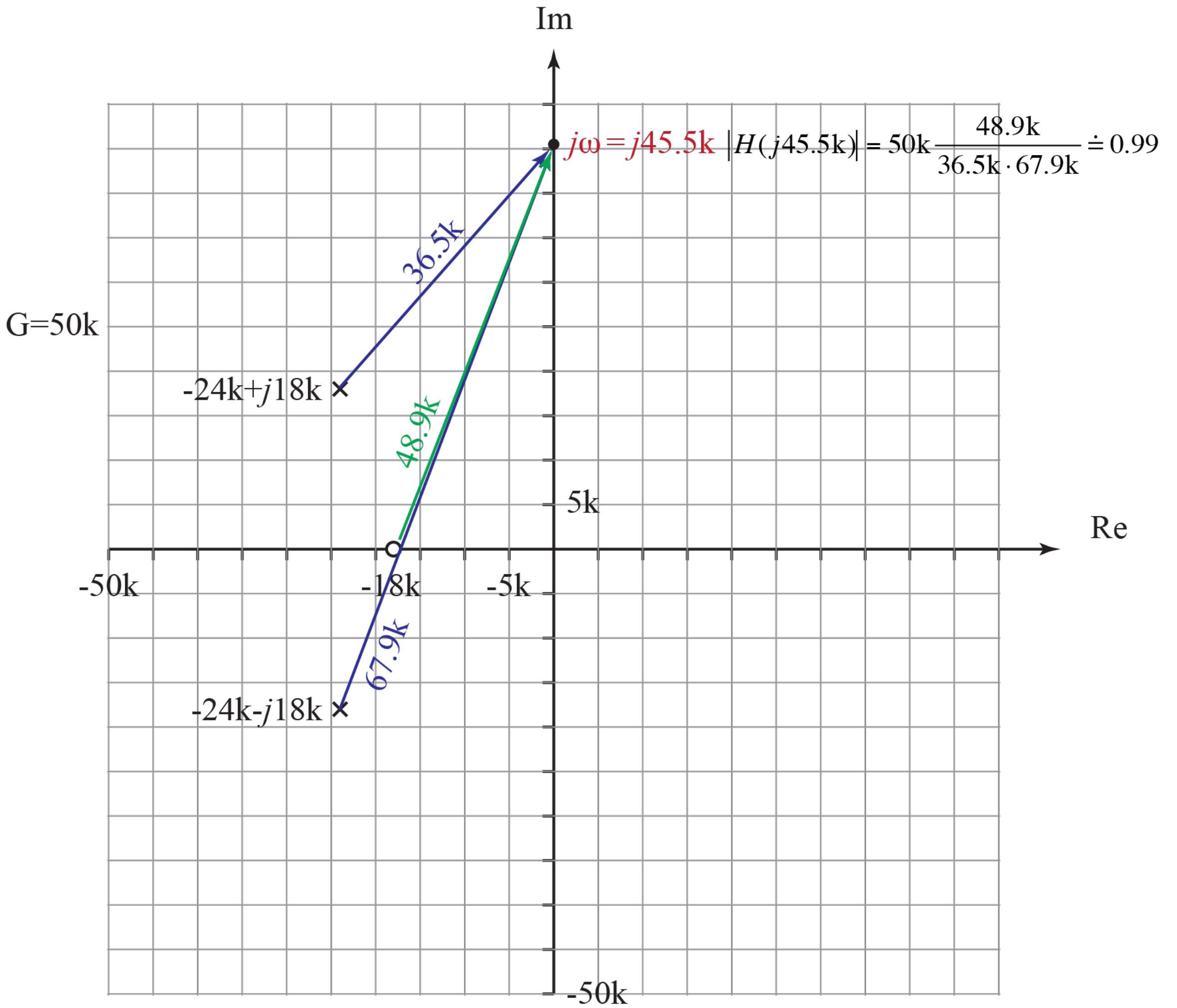


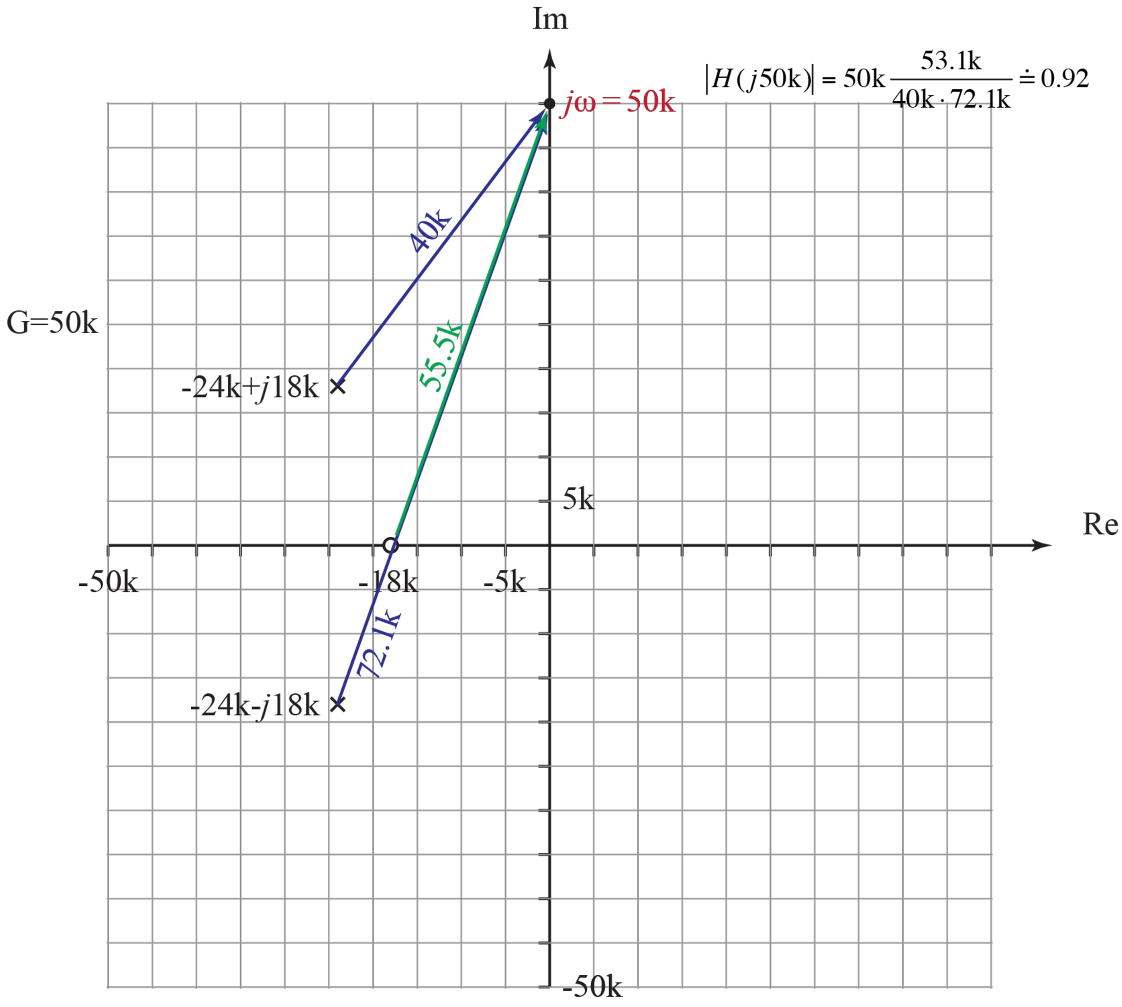

Im
Re
G=50k
-24k+j18k
-24k-j18k
26k
19.7k
35.4k
jω = j8k
5k
-50k
-18k
-5k
|H(j8k)| = 50k 19.7k / 26k · 35.4k ≐ 1.07

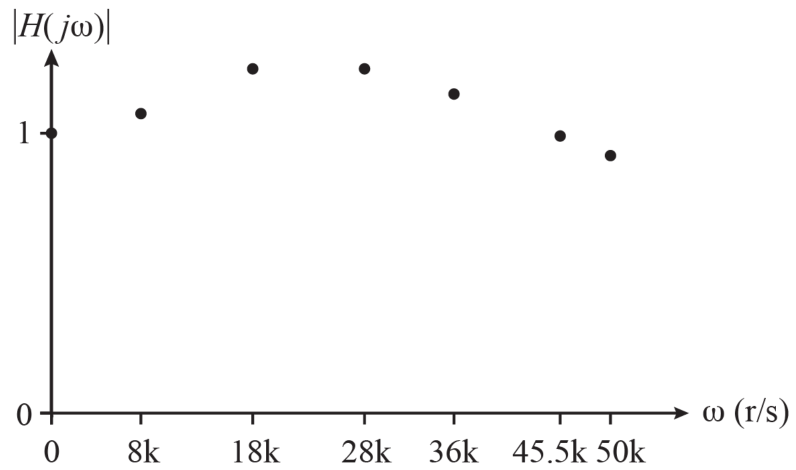

Im
Re
G=50k
-24k+j18k
18k
jω = j18k
|H(j18k)| = 50k 25.5k / (24k · 43.3k) ≐ 1.23
25.5k
5k
-50k
-18k
-5k
43.3k
-24k-j18k
-50k


Im
Re
G=50k
$j\omega = j28\text{k}$
$|H(j28\text{k})| = 50\text{k}\dfrac{33.3\text{k}}{26\text{k}\cdot 51.9\text{k}} \doteq 1.23$
26k
33.3k
51.9k
-24k+j18k
-24k-j18k
5k
-50k
-18k
-5k
-50k


Im
Re
jω = j36k
|H(j36k)| = 50k 40.2k / (30k · 59.1k) ≐ 1.14
G=50k
-24k+j18k
-24k-j18k
30k
40.2k
59.1k
5k
-50k
-18k
-5k
-50k


Im
Re
jω = j45.5k
|H(j45.5k)| = 50k (48.9k)/(36.5k · 67.9k) ≐ 0.99
36.5k
48.9k
67.9k
G=50k
-24k+j18k
-24k-j18k
5k
-50k
-18k
-5k
-50k

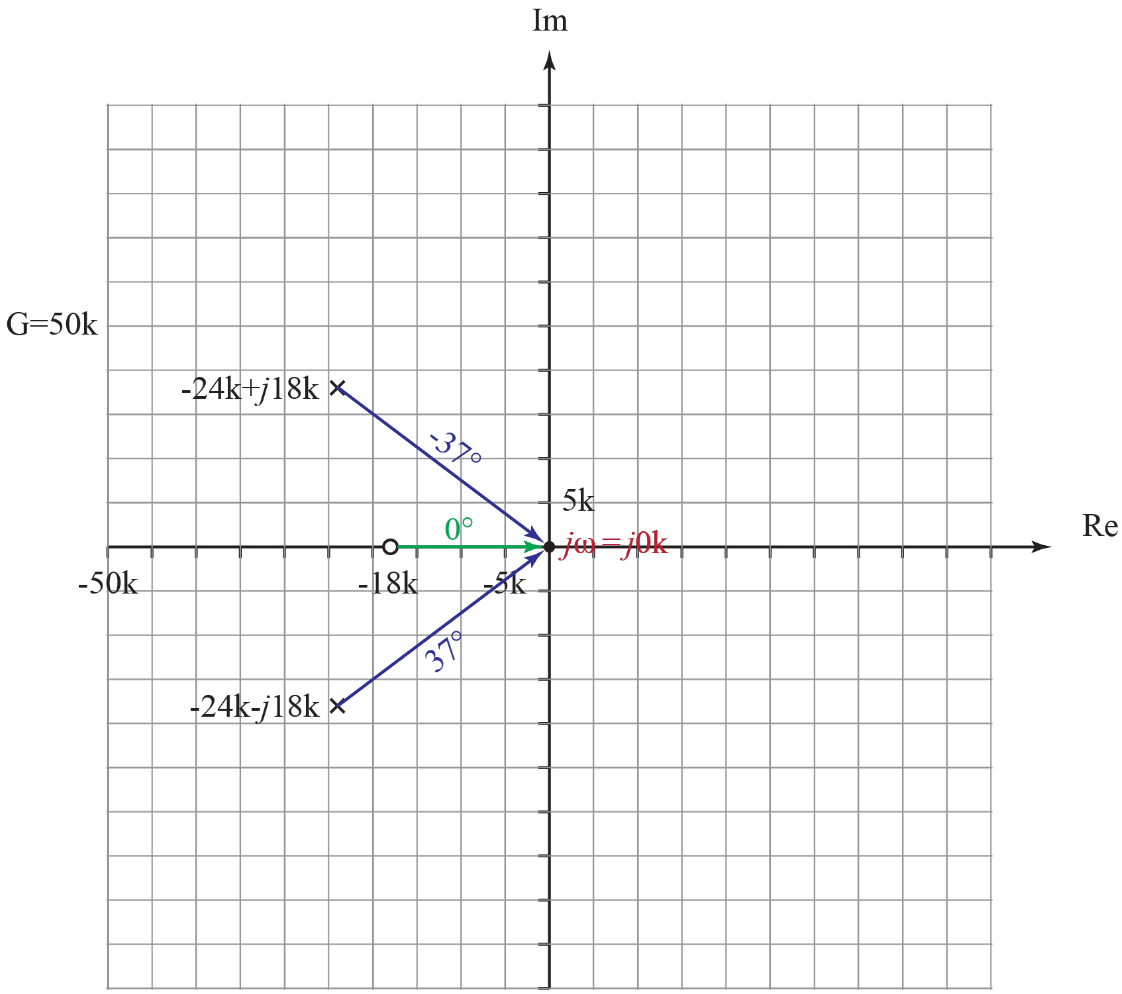

Im
Re
|H(j50k)| = 50k 53.1k / (40k · 72.1k) ≐ 0.92
jω = 50k
G=50k
-24k+j18k
-24k-j18k
40k
55.5k
72.1k
5k
-50k
-18k
-5k
-50k

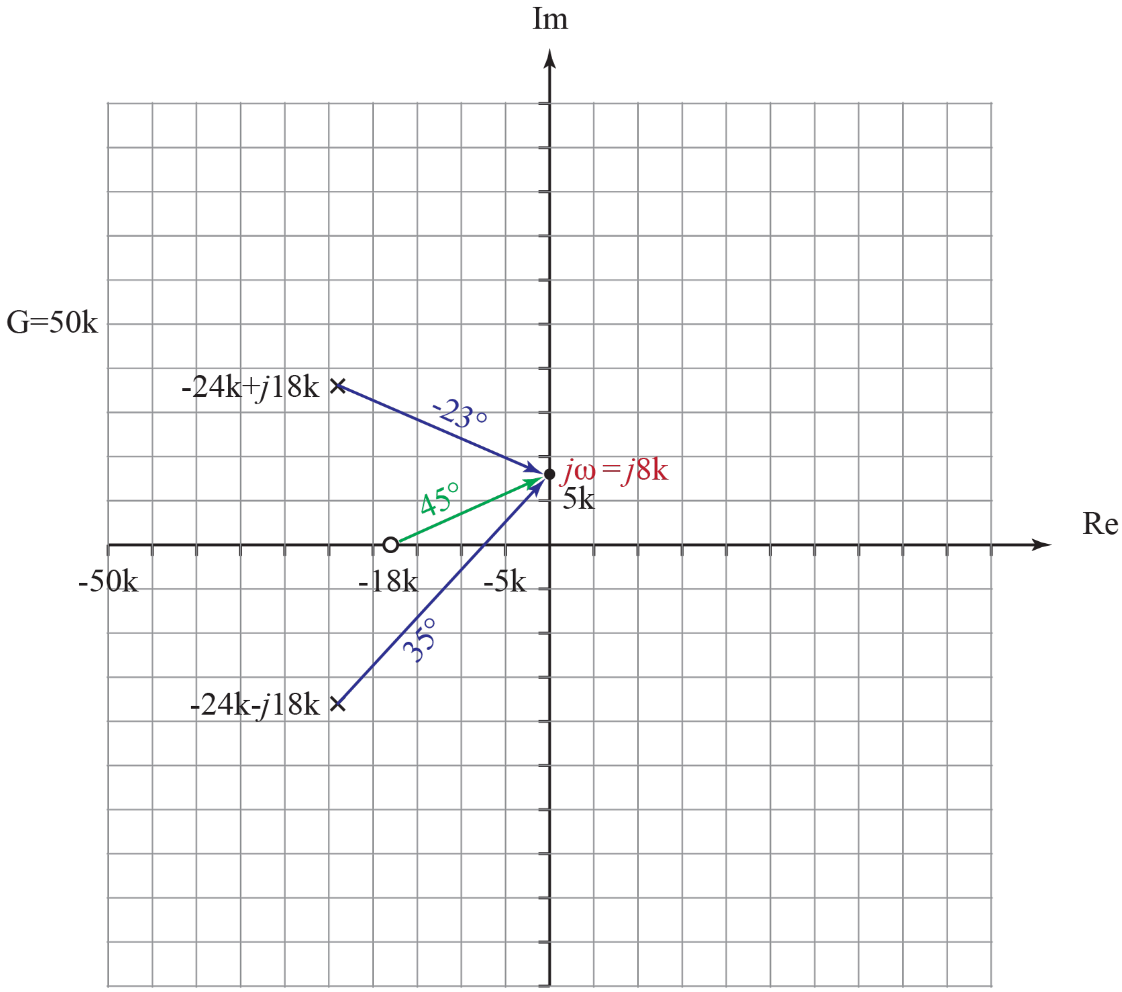

|H(jω)|
1
0
0
8k
18k
28k
36k
45.5k
50k
ω (r/s)

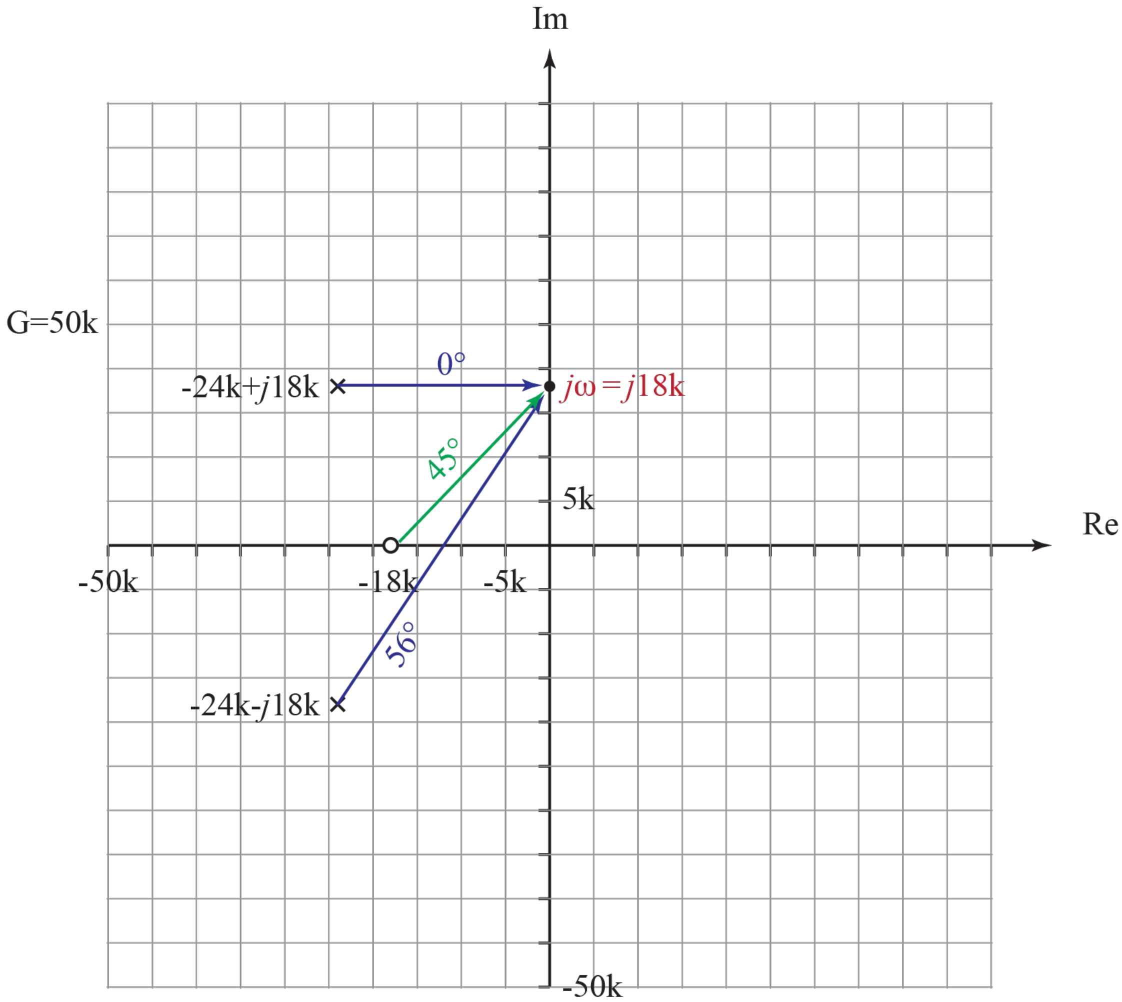
The phase angle of the frequency response is given by the sums and differences of angles from poles and zeros to $s = j\omega$.

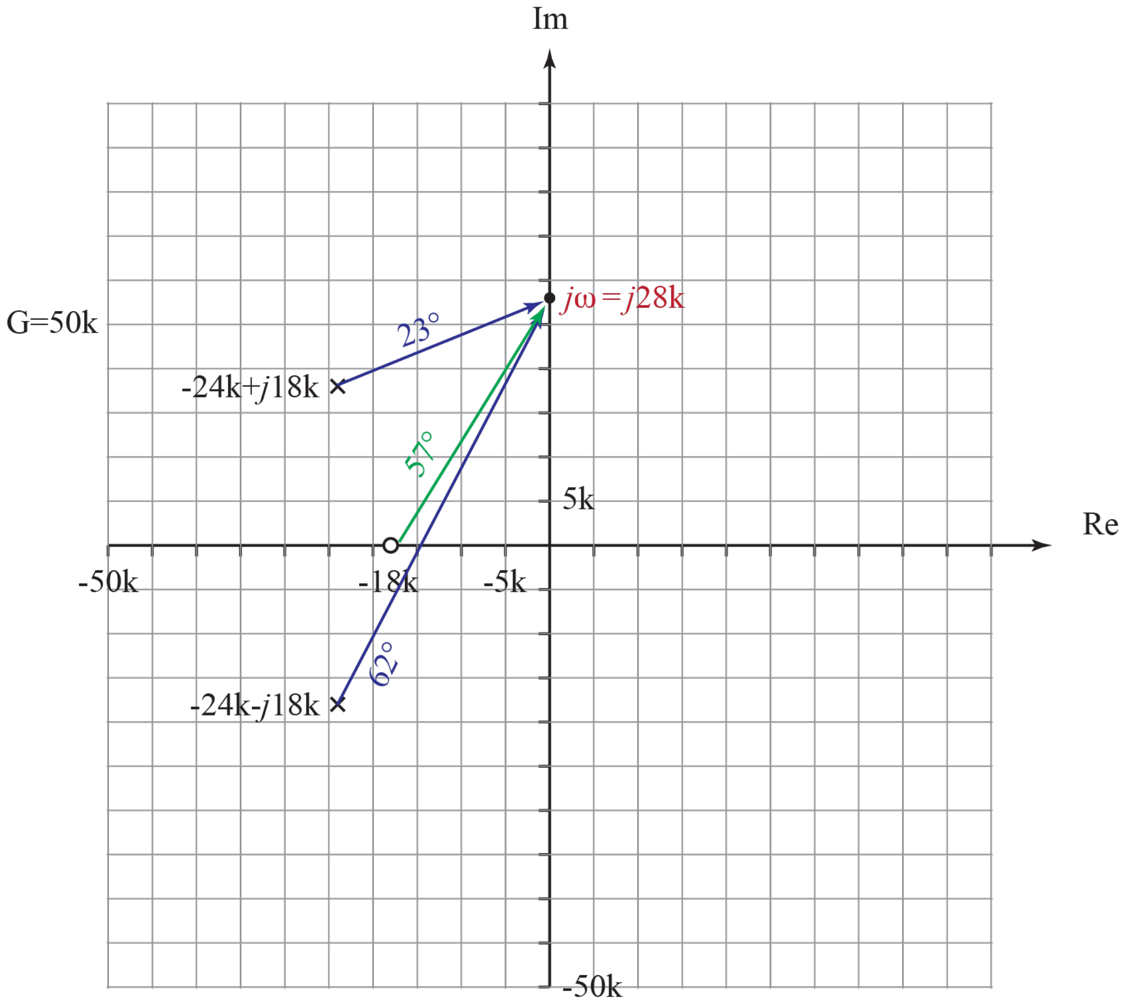
$$\angle H(j\omega) = \angle 50\text{k} + \angle(s - -18\text{k}) - \angle(s - -24\text{k} - j18\text{k}) - \angle(s - -24\text{k} + j18\text{k})$$

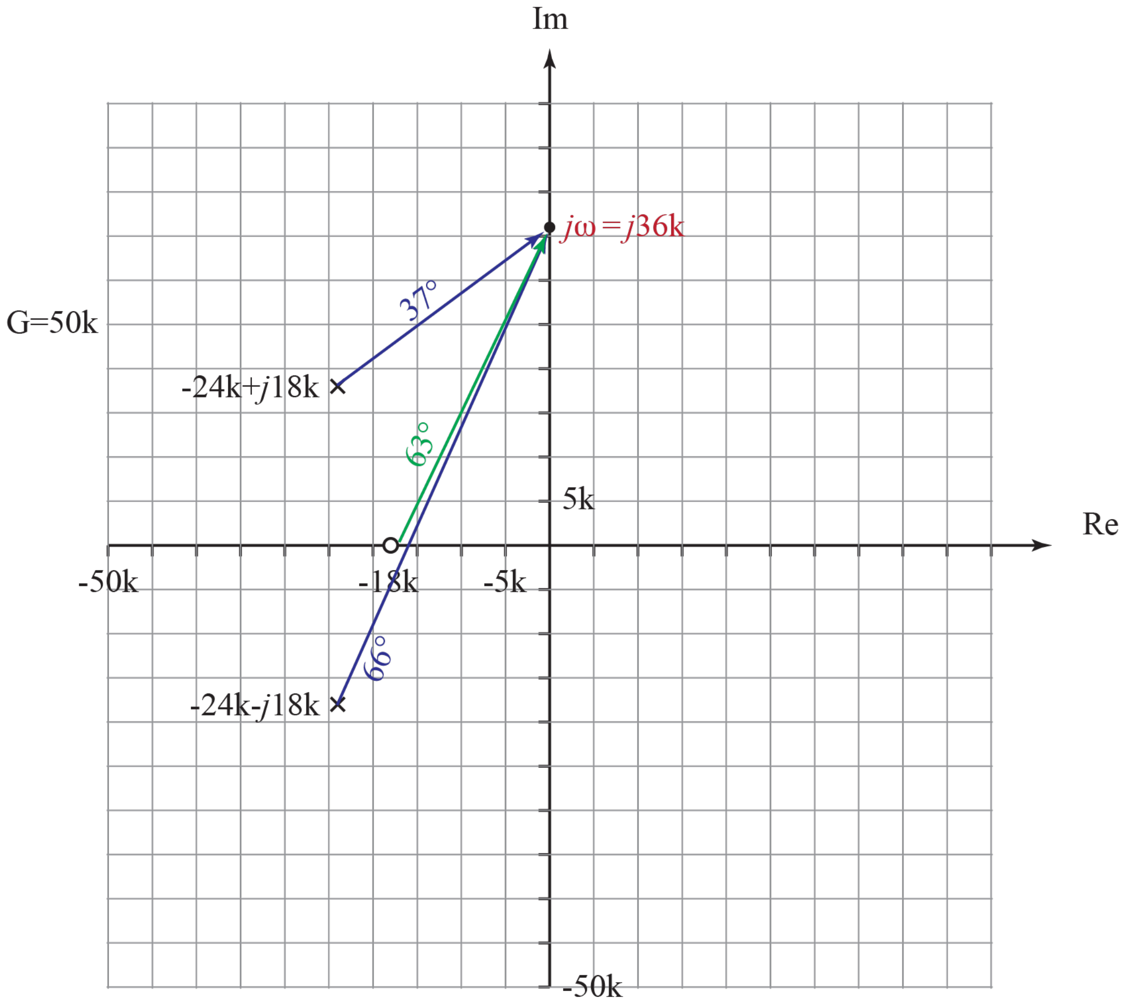
where $s = j\omega$

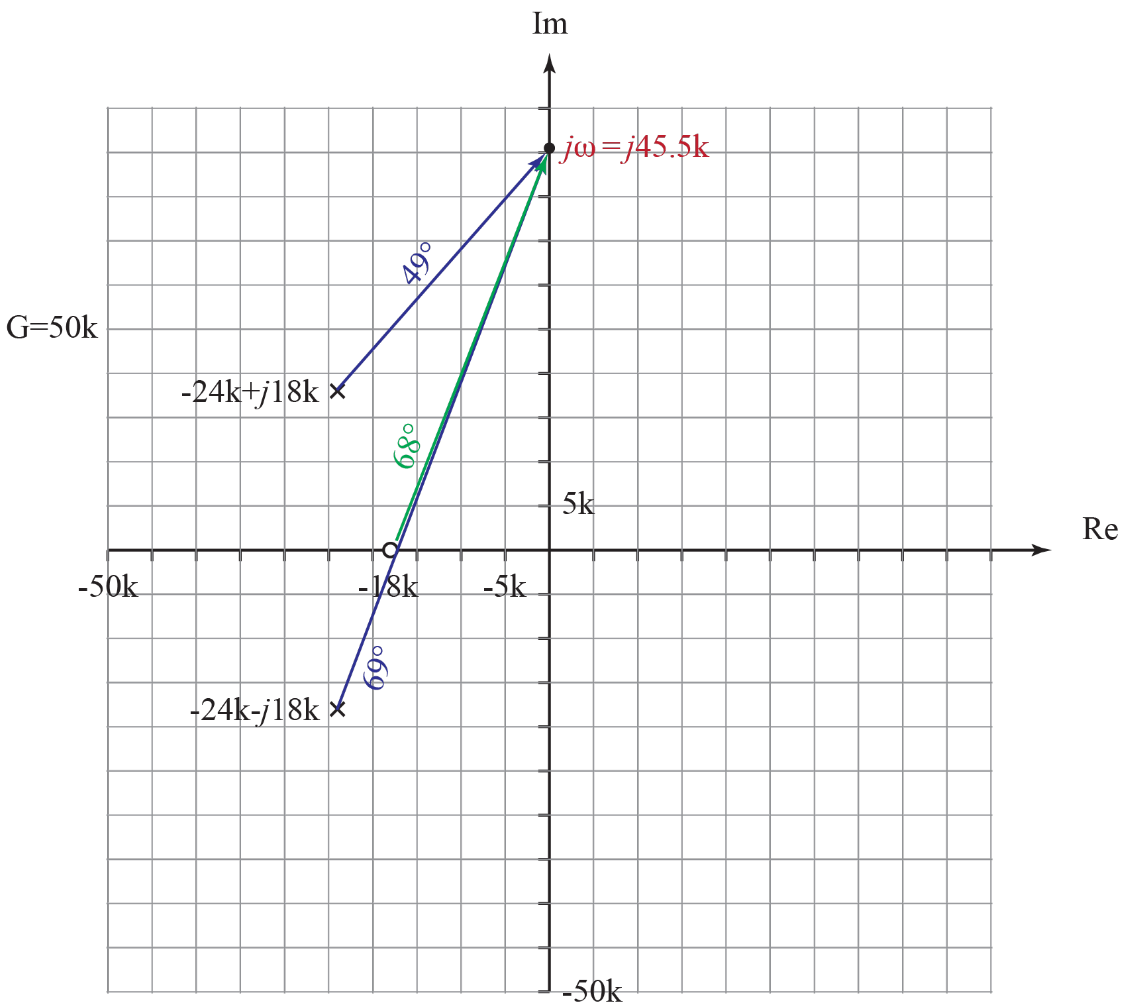


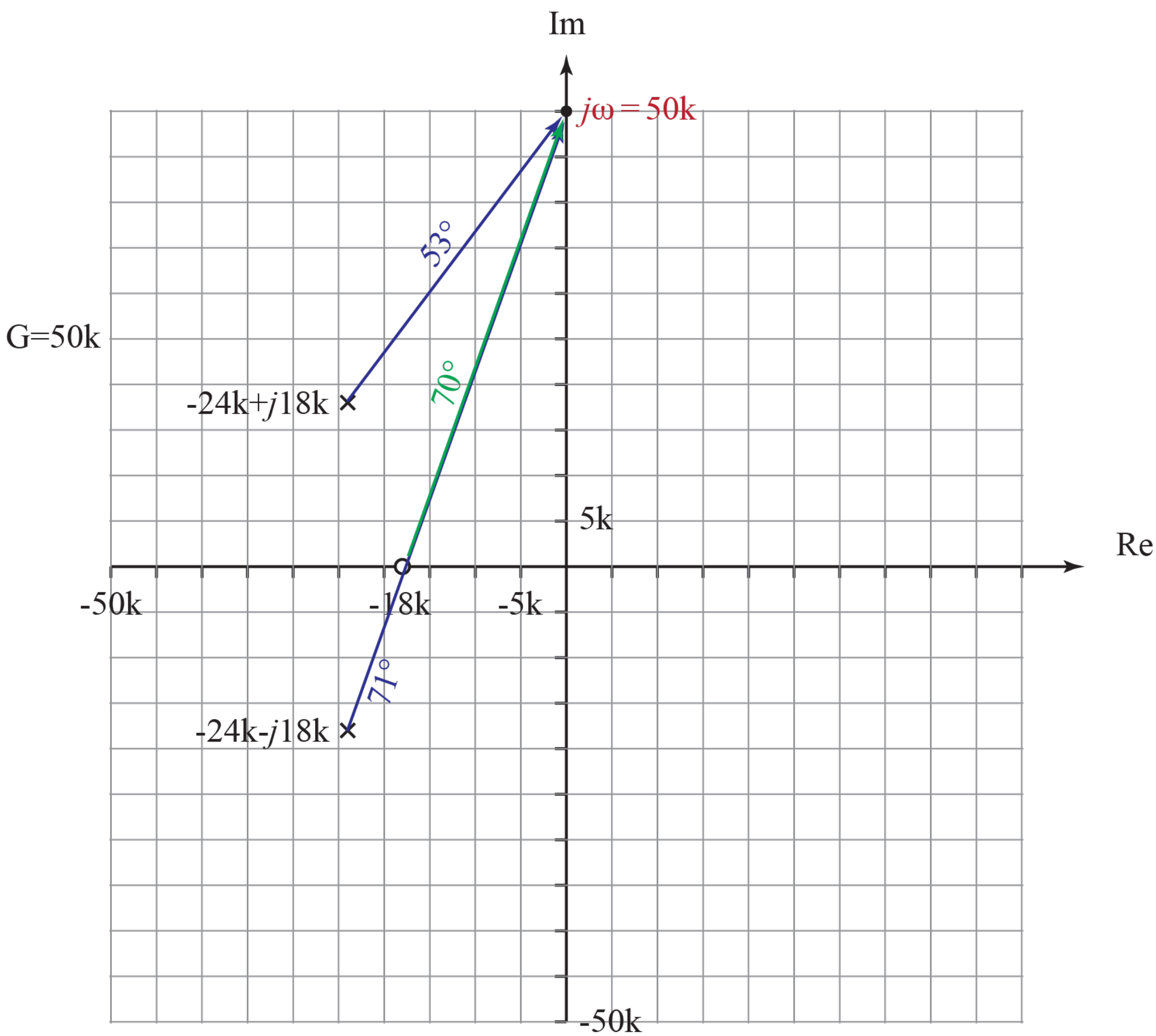

Im
Re
G=50k
-24k+j18k
-24k-j18k
-23°
45°
35°
jω = j8k
5k
-50k
-18k
-5k


Im
Re
G=50k
-24k+j18k
0°
jω = j18k
45°
5k
-50k
-18k
-5k
56°
-24k-j18k
-50k

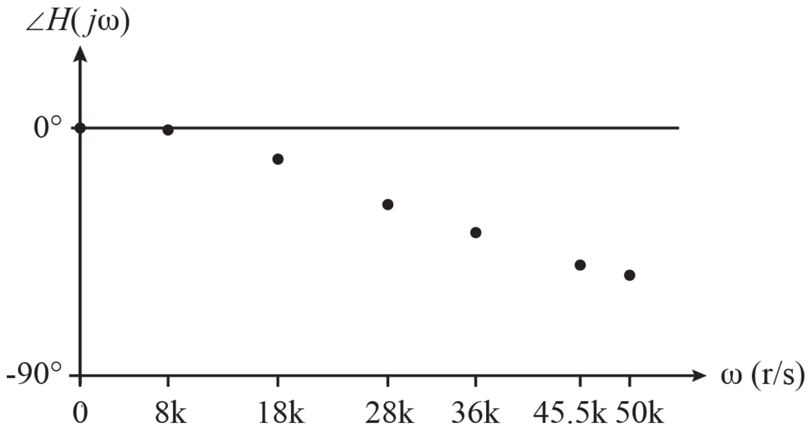

Im
Re
G=50k
$j\omega = j28k$
23°
57°
62°
-24k+j18k
-24k-j18k
5k
-50k
-18k
-5k
-50k


Im
Re
jω = j36k
G=50k
37°
-24k+j18k
63°
5k
-50k
-18k
-5k
66°
-24k-j18k
-50k


Im
Re
jω = j45.5k
49°
G=50k
-24k+j18k
68°
5k
-50k
-18k
-5k
69°
-24k-j18k
-50k



The phase plotted versus frequency:


∠H(jω)
0°
-90°
ω (r/s)
0
8k
18k
28k
36k
45.5k
50k