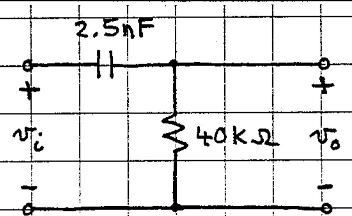
Ref: Nilsson & Riedel, Rev 6, Prob 14.7



a) Find cutoff frequency (Hz) for above high-pass filter.

b) Find $H(j\omega)$ for $\omega = \omega_c$, $0.1\omega_c$, and $10\,\omega_c$.

c) Given $v_i = 800 \cos \omega t$ mV, find $v_o(t)$ for $\omega = \omega_c, 0.1\omega_c, 10\omega_c$.

ans: a) $f_c = 1590$ Hz

b) $H(j\omega_c) = \frac{1}{\sqrt{2}} \angle 45°$, $H(j0.1\omega_c) = 99.5 \angle 84.3°$ m, $H(10\omega_c) = 0.995 \angle 5.71°$

c) $v_o(\omega_c) = 566 \cos(10\text{k}t + 45°)$ mV

$v_o(0.1\omega_c) = 79.6 \cos(1\text{k}t + 84.3°)$ mV

$v_o(10\omega_c) = 796 \cos(100\text{k}t + 5.71°)$ mV

sol'n a) As noted in sol'n to 14.1, the cutoff frequency for a high-pass or low-pass is where $Re[H(j\omega)] = Im[H(j\omega)]$.

Our transfer function $H(j\omega) = \frac{V_o}{V_i}$.

$V_o = V_i \cdot \frac{R}{R + 1/j\omega C}$ V-divider $\therefore H(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + 1/j\omega C}$

$$H(j\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega RC}{1 + j\omega RC} \cdot \frac{1 - j\omega RC}{1 - j\omega RC} = \frac{(\omega RC)^2 + j\,\omega RC}{1 + (\omega RC)^2}$$

Denominator is real, so solve for real part of numerator = imaginary part of numerator.

$$(\omega_c RC)^2 = \omega_c RC \quad \text{or} \quad \omega_c RC = 1 \quad \text{or} \quad \omega_c = \frac{1}{RC}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi RC} = \frac{1}{2\pi\, 40\text{k} \cdot 2.5\text{n}} = \frac{1\text{M}}{2\pi \cdot 100} = \frac{10\text{k}}{2\pi} \text{ Hz}$$

$$f_c = 1590 \text{ Hz}$$

b) Use $H(j\omega)$ from part (a).

$RC = 40k \cdot 2.5n = 100\mu$ (but always cancels)

$$H(j\omega_c) = \frac{\left(\frac{1}{RC}RC\right)^2 + j\frac{1}{RC}RC}{1+\left(\frac{1}{RC}RC\right)^2} = \frac{1+j}{1+1} = \frac{1}{2}+j\frac{1}{2}$$

$$'' = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}\angle\tan^{-1}\frac{1/2}{1/2} = \frac{1}{\sqrt{2}}\angle 45^\circ$$

$$H(j\,0.1\,\omega_c) = \frac{(0.1)^2 + j\,0.1}{1+(0.1)^2} = \frac{0.01+j0.1}{1+0.01} = \frac{0.01+j0.1}{1.01}$$

$$'' = \frac{\sqrt{(0.01)^2+(0.1)^2}}{1.01}\angle\underbrace{\tan^{-1}\frac{0.1}{0.01}}_{\text{can ignore real denominator if it's positive}} = 99.5\angle 84.3^\circ\ \text{m}$$

$$H(j\,10\omega_c) = \frac{10^2+j10}{1+10^2} = \frac{100+j10}{101}$$

$$'' = \frac{\sqrt{100^2+10^2}}{101}\angle\tan^{-1}\frac{10}{100} = 0.995\angle 5.71^\circ$$

c) Phasor $V_o = V_i \cdot H(j\omega)$

$$v_i(t) = 800\cos\omega t\ \text{mV} \overset{P[\,]}{\Rightarrow} V_i = 800\angle 0^\circ\ \text{mV}$$

$$V_o(\omega_c) = 800\angle 0^\circ\ \text{mV}\cdot\underbrace{\frac{1}{\sqrt{2}}\angle 45^\circ}_{H(j\omega_c)} = 566\angle 45^\circ\ \text{mV}$$

$v_o(t)$ for $\omega=\omega_c$ $= 566\cos(\omega t + 45^\circ)$ mV

$V_o(0.1\omega_c) = 800\angle 0^\circ\text{mV}\cdot 99.5\angle 84.3^\circ\text{m} = 79.6\angle 84.3^\circ$ mV

$v_o(t)$ for $\omega = 0.1\omega_c$ $= 79.6\cos(\omega t + 84.3^\circ)$ mV

$V_o(10\omega_c) = 800\angle 0^\circ\ \text{mV}\cdot 0.995\angle 5.71^\circ = 796\angle 5.71^\circ$ mV

$v_o(t)$ for $\omega = 10\omega_c$ $= 796\cos(\omega t + 5.71^\circ)$ mV