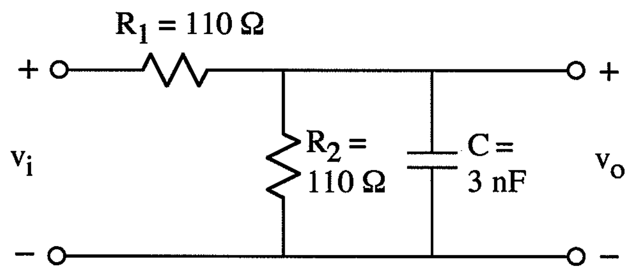
Ex:

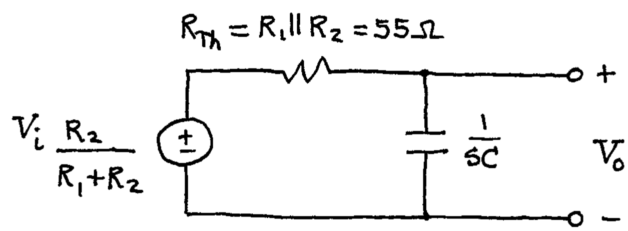


a) Determine the transfer function V_o/V_i. **Hint:** Use a Thevenin equivalent to reduce the two R's to a single R.

b) Plot $|V_o/V_i|$ versus ω.

c) Find the cutoff frequency, ω_c.

sol'n: a) Use a Thevenin equivalent for v_i, R_1, and R_2:



We get the transfer function, $H(s)$, from the voltage-divider formula:

$$V_o(s) = V_i(s)\frac{R_2}{R_1+R_2} \cdot \frac{1/sC}{1/sC + R_{Th}}$$

Divide by sides by $V_i(s)$ to obtain $H(s)$:

$$H(s) \equiv \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1+R_2}\,\frac{1/sC}{1/sC + R_{Th}}$$

A convenient form for $H(s)$ is $H(s) = k \dfrac{1}{1 + jX}$

where $k \equiv$ real constant
$X \equiv$ real term involving ω and component values

In the present case, we multiply top and bottom of $H(s)$ by sC.

$$H(s) = \frac{R_2}{R_1 + R_2} \frac{1}{1 + s R_{Th} C} = \frac{R_2}{R_1 + R_2} \frac{1}{1 + j \omega R_{Th} C}$$

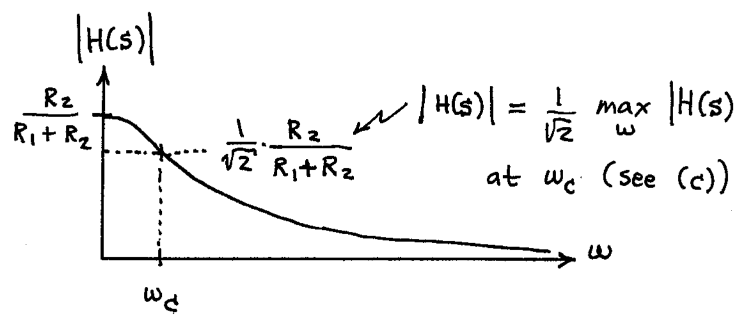
b) Since $\dfrac{1}{sC} = \dfrac{1}{0} = \infty$ @ $\omega = 0$, $V_o = V_i \dfrac{R_2}{R_1 + R_2}$.

$\therefore |H(s = j0)| = \dfrac{R_2}{R_1 + R_2}$ for $\omega = 0$

Since $\dfrac{1}{sC} = \dfrac{1}{\infty} = 0$ @ $\omega = \infty$, we effectively have $C =$ wire. Thus, $V_o = 0$.

$\therefore |H(s = j\infty)| = 0$ for $\omega = \infty$

Between $\omega = 0$ and $\omega = \infty$, the C starts to become a short circuit and $|H(s)|$ decreases.



c) ω_c is the value of ω where $|H(s)|$ is $\frac{1}{\sqrt{2}}$ times $\max_{\omega} |H(s)|$.

In other words, $$\frac{|H(j\omega_c)|}{\max_{\omega}|H(s)|} = \frac{1}{\sqrt{2}}.$$

In the present case, we have

$$\frac{|H(j\omega_c)|}{\max_{\omega}|H(s)|} = \frac{\frac{R_2}{R_1+R_2}\frac{1}{1+j\omega_c R_{Th}C}}{\frac{R_2}{R_1+R_2}} = \frac{1}{\sqrt{2}}$$

or $$\frac{1}{1+j\omega_c R_{Th}C} = \frac{1}{\sqrt{2}}$$

or $$1+j\omega_c R_{Th}C = \sqrt{2}$$

or $$\sqrt{1^2+(\omega_c R_{Th}C)^2} = \sqrt{2}$$

or $$1+(\omega_c R_{Th}C)^2 = \pm 2$$

Since $(\omega_c R_{Th}C)^2 > 0$, we must have

$$1+(\omega_c R_{Th}C)^2 = 2$$

or $(\omega_c R_{Th}C)^2 = 1$ or $\omega_c R_{Th}C = \pm 1$

We must have $\omega_c R_{Th}C = +1$ or $\omega_c = \frac{1}{R_{Th}C} = \frac{1}{55\Omega \cdot 3nF} \doteq 6\,Mr/s$