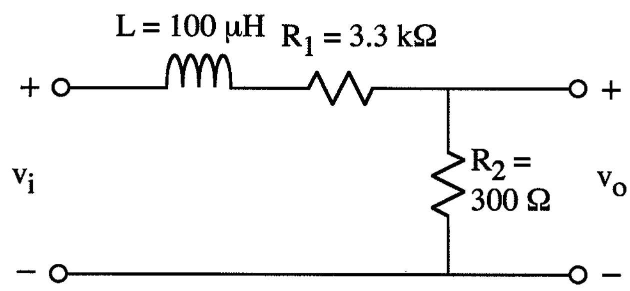
Ex:

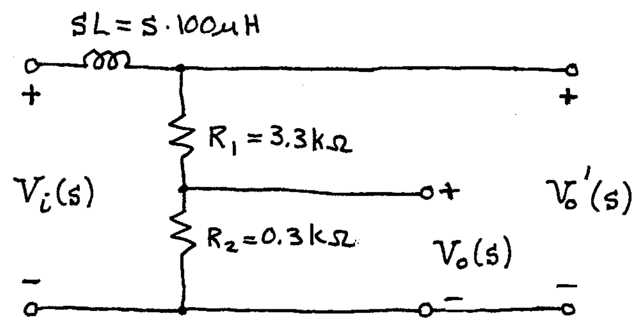


a) Determine the transfer function V_o/V_i. **Hint:** Suppose the output were tapped from the point between L and R_1. Then use a voltage divider.

b) Plot $|V_o/V_i|$ versus ω.

c) Find the cutoff frequency, ω_c.

sol'n: a) Consider tapping the output from between L and R_1. Then use a V-divider to relate v_o to the voltage between L and R_1.



$$V_o(s) = V_o'(s)\frac{R_2}{R_1+R_2}$$

Using the V-divider formula, we have

$$V_o' = V_i(R_1+R_2)/(R_1+R_2+sL)$$

Thus, $H(s) = \dfrac{V_o(s)}{V_i(s)} = \dfrac{R_2}{R_1+R_2} \dfrac{R_1+R_2}{R_1+R_2+sL}$

$$H(s) = \frac{R_2}{R_1+R_2+sL}$$

b) $|V_o/V_i| = |H(s)|$

$sL = 0$ at $\omega = 0 \Rightarrow L = \text{wire}$ at $\omega = 0$

Thus, for $\omega = 0$, we have a simple V-divider.

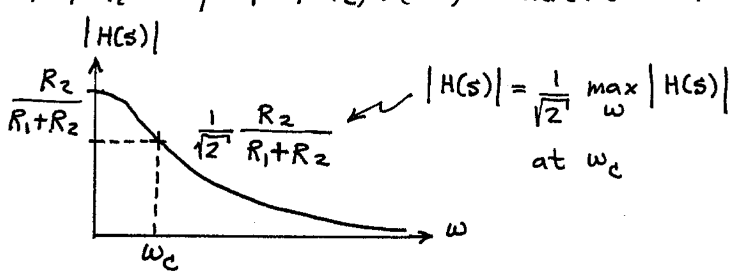
$$|H(s = j0)| = \frac{R_2}{R_1+R_2}$$

$sL \to \infty$ as $\omega \to \infty \Rightarrow L = \text{open}$ as $\omega \to \infty$

Thus, the output is disconnected from the input and $|H(s)| \to 0$ as $\omega \to \infty$.

$|H(s)|$ decreases as ω increases since

$|R_1+R_2+sL| = \sqrt{(R_1+R_2)^2 + (\omega L)^2}$ increase with ω.



We find ω_c below.

c) ω_c is the ω where $|H(s)|$ is reduced by a factor of $\sqrt{2}$ relative to $\max_\omega |H(s)|$.

Thus, $\left|H(s=j\omega_c)\right| = \frac{1}{\sqrt{2}} \max_\omega |H(s)| = \frac{1}{\sqrt{2}} \frac{R_2}{R_1+R_2}$,

or $\left|\frac{R_2}{R_1+R_2+j\omega_c L}\right| = \frac{1}{\sqrt{2}} \frac{R_2}{R_1+R_2}$.

We write $H(s)$ in the form $k\frac{1}{1+jX}$

where $k \equiv$ real constant, $X \equiv$ real expression

$$H(s) = \frac{R_2}{R_1+R_2} \frac{1}{1+j\omega_c \frac{L}{R_1+R_2}}$$

$$|H(s)| = \frac{R_2}{R_1+R_2} \cdot \frac{1}{\left|1+j\,\omega_c \frac{L}{R_1+R_2}\right|}$$

Thus, we have $\frac{\cancel{R_2}}{\cancel{R_1+R_2}} \frac{1}{\left|1+j\frac{L}{R_1+R_2}\omega_c\right|} = \frac{\cancel{R_2}}{\cancel{R_1+R_2}} \frac{1}{\sqrt{2}}$

So $\left|1+j\frac{L\omega_c}{R_1+R_2}\right| = \sqrt{2}$.

The solution is $1+j\frac{L\omega_c}{R_1+R_2} = 1 \pm j$,

since $|1 \pm j| = \sqrt{2}$. We must have $\frac{L\omega_c}{R_1+R_2} = 1$.

Thus, $\omega_c = \frac{R_1+R_2}{L} = \frac{3.3\text{k}\Omega + 0.3\text{k}\Omega}{100\mu\text{H}}$

$$\omega_c = \frac{3.6\text{k}\Omega}{100\mu\text{H}} = 36 \text{ M r/s}$$