EX: Make calculations of |Z| for enough values of ω that you can plot a good graph of |Z| versus ω for each of the circuits shown in Figs. 1 and 2. Then vary the values of the components, and make enough further calculations that you can get a qualitative feel for how the size and shape of the curve depends on the parameters, without making numerical calculations. For example, if |Z| versus ω is given for the circuit shown in Fig. 3, explain how the curve would change if R were doubled, with nothing else changed. Explain how the curve would change if L were doubled, with nothing else changed.

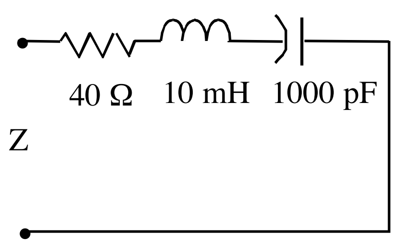

Fig. 1

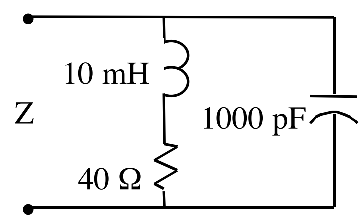

Fig. 2

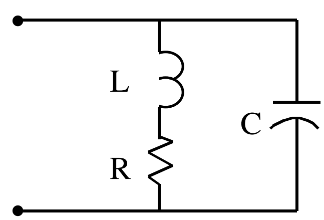

Fig. 3

ANS: Fig. 1) plot of |z| will be bowl-shaped with a minimum value of R at ω_o.

Fig. 2) $|z| = R = 40\ \Omega$ at $\omega = 0$.

|z| increases to a resonant peak of (approximate) height $kR = 6.25\ \text{k} \cdot 40\ \Omega = 250\ \text{k}\Omega$. The resonant frequency is $1/\sqrt{(LC)} = 1\ \text{M}/\sqrt{10}$ rad/s.

|z| then decreases toward 0 as $\omega \to \infty$.

Fig. 3) Double R: $|z|$ starts at twice the height but the resonant peak height is only half as high. The frequency of the resonant peak is unaffected. Double L, then the curve for $|z|$ still starts at height R but the resonant peak height is about twice as high. The frequency of the resonant peak is lowered by a factor of $\sqrt{2}$.

SOL'N: Fig. 1) $z = R + j\omega L + -j/(\omega C) = R + j[\omega L - 1/(\omega C)]$

$|z|^2 = R^2 + [\omega L - 1/(\omega C)]^2$

We may visualize $|z|$ as the length of the hypotenuse of a right triangle in the complex plane with sides of length R and $[\omega L - 1/(\omega C)]$.

At the resonant frequency of the L and C, $\omega_o{}^2 = 1/(LC)$, the L and C act like a wire: $\omega L - 1/(\omega C) = 0$. Thus, $z = R$ at ω_o.

For z above or below ω_o, the magnitude of $[\omega L - 1/(\omega C)]$ increases. Thus, our plot of $|z|$ will be bowl-shaped with a minimum value of R at ω_o.

Fig. 2) $z = (R + j\omega L) \parallel 1/(j\omega C)$

Consider the qualitative behavior at key frequencies: (Note that the lower magnitude impedance dominates in a parallel configuration.)

At $\omega = 0$, $R + j\omega L = R$ and $1/(j\omega C) = \infty$.

Thus, $z = R$.

At $\omega = R/L$, $R + j\omega L = R(1+j)$ and $1/(j\omega C) = -jR[(L/R)/(RC)]$.

Thus, $z = R[(1 + j) \parallel -j(L/R)/(RC)]$, and z will behave more like the smaller of the parallel terms. If $L/R = RC$, however, we will have $z = R[(1 + j) \parallel -j] = R(1 - j)$. For the component values given, $L/R = 1/4$ ms, $RC = 40$ ns, and $(L/R)/(RC) = 6.25$ k. It follows that $z \approx R(1+j)$ at $\omega = R/L$ for this circuit.

As $\omega \to \infty$, $R + j\omega L \to R + j\infty$ and $1/(j\omega C) \to 0$.

Thus, $z \to 0$.

Now we examine the behavior versus frequency more carefully. Our calculations are driven by the thought that the resonance of the L and C may cause the L and C to become almost an open circuit. The R, however, will affect the resonance.

To simplify the algebra, we use normalized (and unitless) frequency $\omega_n = \omega/(R/L)$. We have $z = R[(1 + j\omega_n) \,\|\, -j(L/R)/(RC\omega_n)]$.

Define $k \equiv (L/R)/(RC)$. Then we have $z = R[(1 + j\omega_n) \,\|\, -jk/\omega_n]$ or $z = R[(-jk/\omega_n + k)/(1 + j[\omega_n - k/\omega_n])]$.

Define $\omega_{no} \equiv \sqrt{k}$. At $\omega_n = \omega_{no}$, we have $\omega = \omega_o \equiv 1/\sqrt{(LC)}$, $\omega_{no} = k/\omega_{no}$, and $z = R[-j\omega_{no} + k] = R[-j\sqrt{k} + k]$.

If $k << 1$, then $z \approx Rk << R$ and the L and C do not act like an open circuit at resonance. Instead, the circuit acts like the C is nearly shorted.

If $k >> 1$, then $z \approx Rk >> R$ and the L and C almost act like an open circuit at resonance. This case applies to Fig. 2.

Now we have enough information to describe the shape of $|z|$ versus ω as ω goes from 0 to ∞:

$|z| = R = 40\ \Omega$ at $\omega = 0$.

$|z|$ increases to a resonant peak of (approximate) height $kR = 6.25\text{ k} \cdot 40\ \Omega = 250\text{ k}\Omega$. The resonant frequency is $1/\sqrt{(LC)} = 1\text{ M}/\sqrt{10}$ rad/s.

$|z|$ then decreases toward 0 as $\omega \to \infty$.

Fig. 3) Based on calculations for Fig. 2, if we double R then the curve for $|z|$ starts at twice the height but the resonant peak height of Rk is only half as high (since Rk behaves like 1/R). The frequency of the resonant peak is unaffected.

If we double L, then the curve for $|z|$ still starts at height R but the resonant peak height of Rk is about twice as high (since k is proportional to L). The frequency of the resonant peak is lowered by a factor of $\sqrt{2}$.

Note: These results only apply to the case $k >> 1$. If R becomes too large, then our approximations break down (and the resonant peak may disappear entirely).