ex: Hermite polynomials [1]

$$H_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n} e^{-x^2/2}$$

Note: Hermite polynomials arise in probability. Observe the term $e^{-x^2/2}$ that is found in the standard normal distribution, $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

n	$H_n(x)$
0	1
1	x
2	x^2-1
3	x^3-3x
4	x^4-6x^2+3

Orthogonality:

Domain $D = (-\infty, \infty)$ (possible values for x)

$w(x) = e^{-x^2/2}$ (weighting function)

$$\langle H_m(x), H_n(x) \rangle \equiv \int_{-\infty}^{\infty} H_m(x) H_n(x) w(x)\, dx = n! \sqrt{2\pi}\, \delta_{mn}$$

(inner product)

where $\delta_{mn} = \begin{cases} 1 & m=n \\ 0 & m \neq n \end{cases}$

Note: Hermite polynomials are orthogonal but not orthonormal. Scaling by $\sqrt{\frac{1}{n!\sqrt{2\pi}}}$ makes them orthonormal

ref: [1] Wikipedia – Hermite polynomials