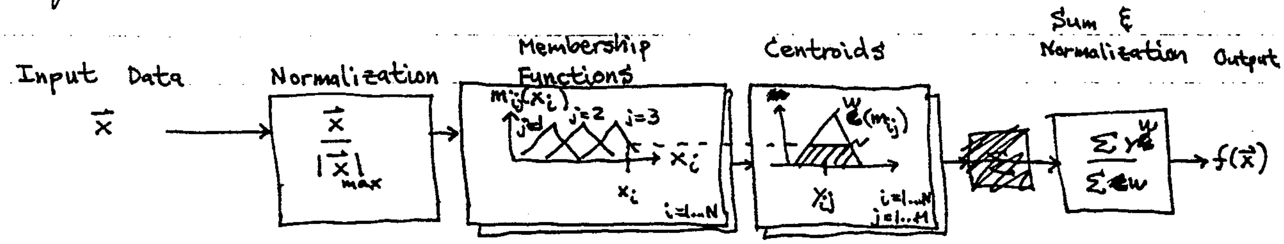
Neil E Cotter ref: Bart Kosko Neural Networks and Fuzzy Systems Prentice Hall: Englewood Cliffs, NJ 1992.

13 [illegible]r 1994 note: Fuzzy logic is a method of interpolating between values located on a fixed grid.

Neil E Cotter 16 Mar 1994

block diagram:

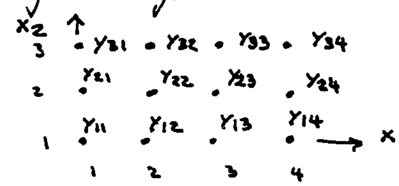


Fuzzy ~~logic~~ Associative Memory (FAM)

~~[illegible]~~ *tool:* Fuzzy logic is based on ~~the~~ a weighted-sum interpolation approach:

1) Define desired system output values for certain input data values. For FAM's we consult an expert system operator for the output values, and we locate ~~them~~ the input values on a rectangular grid.

ex:



For $(x_1, x_2) = (1,1)$ we get $y_{11} = 3.4$

Note: diagram does not show y values. Our view is like looking down on poles of different heights. The heights are y values.

2) Define a membership and centroid function whose value, $w_{ij}(m_{ij}(\vec{x}))$, ~~that~~ indicates how much of each y_{ij} value should be present in the network output $f(\vec{x})$ for a given input data vector $\vec{x}$.

In other words, each y_{ij} gets to vote on how much the output value should look like ~~[illegible]~~ themselves. The voting share is greater for y_{ij} if input $\vec{x}$ is close to the grid point for y_{ij}.

For now, think of membership and centroid as one function.

Neil E. Cotter
13 [illegible]r 1994
Neil E. Cotter
16 Mar 1994

3) Normalize the weighted sum by dividing by the sum of all the weights:

$$f(\vec{x}) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M} y_{ij}\, w_{ij}(m_{ij}(\vec{x}))}{\sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}(m_{ij}(\vec{x}))}$$

4) Design the membership function m_{ij} in such a way that its value is one (1) for $\vec{x} = x_{ij}$, i.e. we want $f(\vec{x}_{ij}) = y_{ij}$ and x_{ij} has 100% membership for y_{ij}.

Also design the membership function m_{ij} to be zero (0) at grid points other than $\vec{x}_{ij}$. Thus, at $\vec{x}_{ij}$ we have only y_{ij} contributing to the output:

$$f(\vec{x}_{ij}) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M} y_{ij}\, w_{ij}(m_{ij}(\vec{x}))}{\sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}(m_{ij}(\vec{x}))} = \frac{y_{ij}\, w_{ij}(m_{ij}(x_{ij}))}{w_{ij}(m_{ij}(x_{ij}))} = y_{ij}$$

Note: we have used i,j in two different ways — as dummy variables for Σ's and as a specific ~~set~~ pair of ~~indices~~ index values for a particular $\vec{x}_{ij}$.

moral: ~~the~~ We have $f(\vec{x}_{ij}) = y_{ij}$ as we would hope; we have produced the outputs specified by the expert, and we have continuous interpolation in between.