Neil E. Cotter
16 1994

def: membership function of $\overline{\tilde{A}}$ = complement of $\tilde{A}$ $\equiv$

$$m_{\overline{\tilde{A}}}(x) = 1 - m_{\tilde{A}}(x)$$

ex: If $\tilde{C} = \tilde{A} \tilde{\wedge} \tilde{B}$ (fuzzy AND) $\tilde{A}, \tilde{B}$ as in fuzzy AND example
then $\overline{\tilde{C}} = \tilde{A}^- \tilde{\vee} \tilde{B}^-$ (fuzzy OR) where $\tilde{A}^- = \tilde{A}$ for $1-x_1$ etc

pf: In the examples for fuzzy AND and fuzzy OR compared to digital AND and OR we see that the fuzzy AND surface + fuzzy OR surface = 1 for every (x_1, x_2).
for $(1-x_1) \tilde{\vee} (1-x_2)$

In other words, $m_{\tilde{A}^- \tilde{\vee} \tilde{B}^-} = 1 - m_{\tilde{A} \tilde{\wedge} \tilde{B}}$.

$$\therefore \quad \tilde{A}^- \tilde{\vee} \tilde{B}^- = \overline{\tilde{A} \tilde{\wedge} \tilde{B}}$$

ex: Is it ever true that if $\tilde{c} = \tilde{A} \tilde{\wedge} \tilde{B}$ then $\overline{\tilde{C}} = \tilde{A} \tilde{\vee} \tilde{B}$?

We have $m_{\tilde{c}} = \min(m_{\tilde{A}}(x), m_{\tilde{B}}(x)) = m_{\tilde{A} \tilde{\wedge} \tilde{B}}$

$$m_{\tilde{A} \tilde{\vee} \tilde{B}} = \max(m_{\tilde{A}}(x), m_{B}(x))$$

If the min() picks $m_{\tilde{A}}$ for a given x, then the max() picks $m_{\tilde{B}}$ and vice versa.

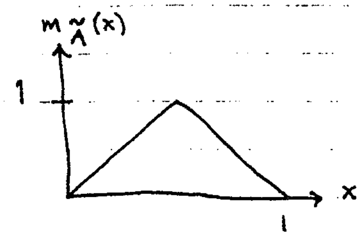
Thus $m_{\tilde{A} \tilde{\wedge} \tilde{B}} + m_{\tilde{A} \tilde{\vee} \tilde{B}} = m_{\tilde{A}}(x) + m_{\tilde{B}}(x)$ always.

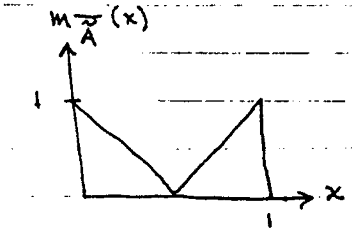
So if $m_{\tilde{A}}(x) + m_{\tilde{B}}(x) = 1$ for every x then we have satisfied the condition that $\tilde{C} = \tilde{A} \tilde{\wedge} \tilde{B}$ and $\overline{\tilde{C}} = \tilde{A} \tilde{\vee} \tilde{B}$.

But this means that $\tilde{B} = \overline{\tilde{A}}$.

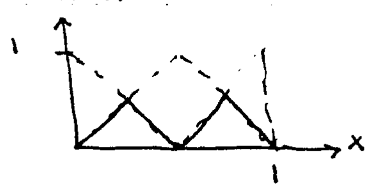
Let's check this with an example.

Neil E Cotter
16 Mar 1994

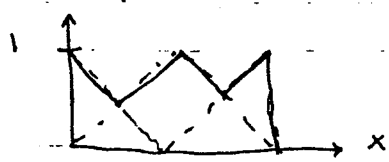




$$m_{\tilde{A} \wedge \bar{\tilde{A}}}(x) = \min(m_{\tilde{A}}(x), m_{\bar{\tilde{A}}}(x))$$

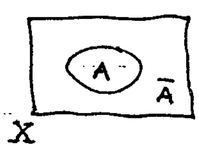


$$m_{\tilde{A} \vee \bar{\tilde{A}}}(x) = \max(m_{\tilde{A}}(x), m_{\bar{\tilde{A}}}(x))$$



We see that these two plots sum to 1. ✓

Note that for regular set theory we have



$$A \wedge \bar{A} = 0$$
$$A \vee \bar{A} = X = \overline{A \wedge \bar{A}}$$

So $A \wedge \bar{A} + A \vee \bar{A} = X$.

Thus, a similar notion holds in regular set theory.