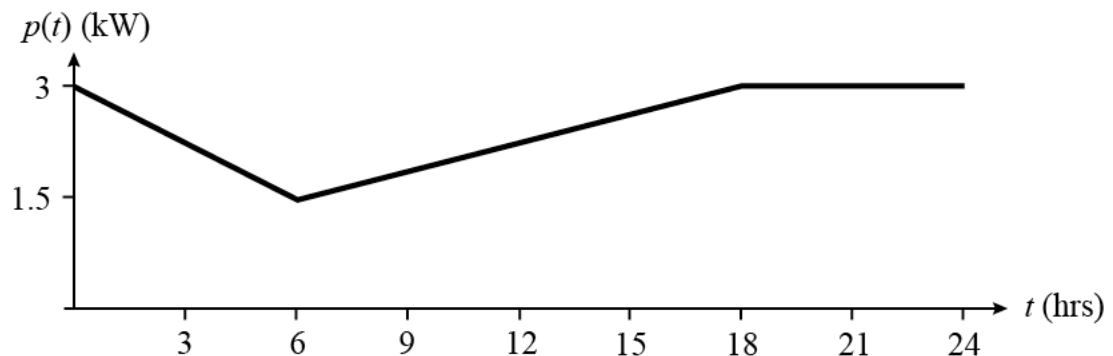


TOOL: The integral of a curve between two points is the area under the curve between the two points.

TOOL: The integral of a curve from zero to t is a function of t and is the area under the curve between zero and each time t . One may imagine that the area is bounded by a vertical line at time zero, a vertical line at t , the curve itself, and the t -axis. The vertical line at t moves to the right as time increases.

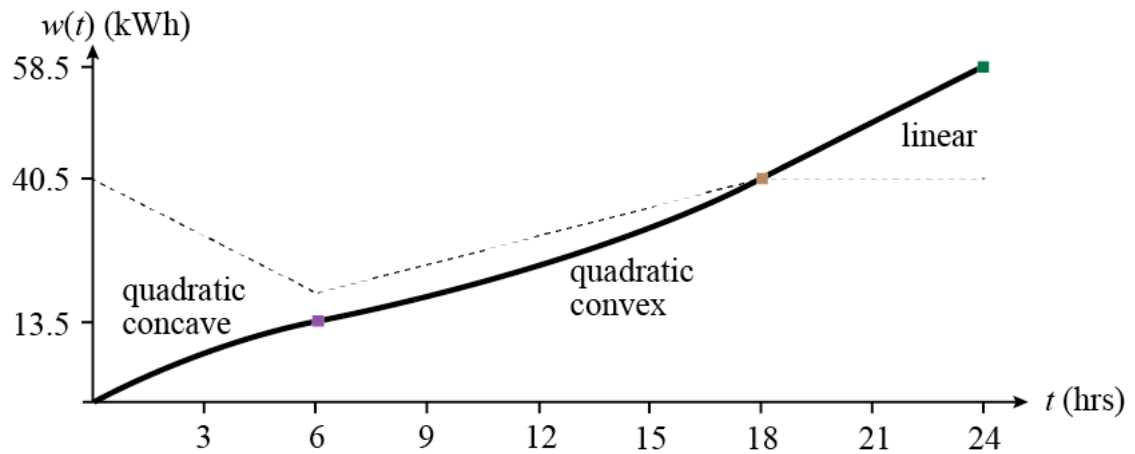
TOOL: The area under a curve may often be calculated (or at least estimated) by breaking the area up into simple geometric shapes like squares, rectangles, and circles whose areas are easy to compute.

Ex: Estimate the energy, w , consumed by a home, if the home's power consumption versus time, $p(t)$, is the function shown below. The equation relating energy to power is $w = \int_0^t p(t) dt = \int_0^t v(t)i(t) dt$. Hint: the integral between time 0 and time t is just the area under the curve between time 0 and time t . Break the area into simple geometric shapes, and sum the areas of the shapes.

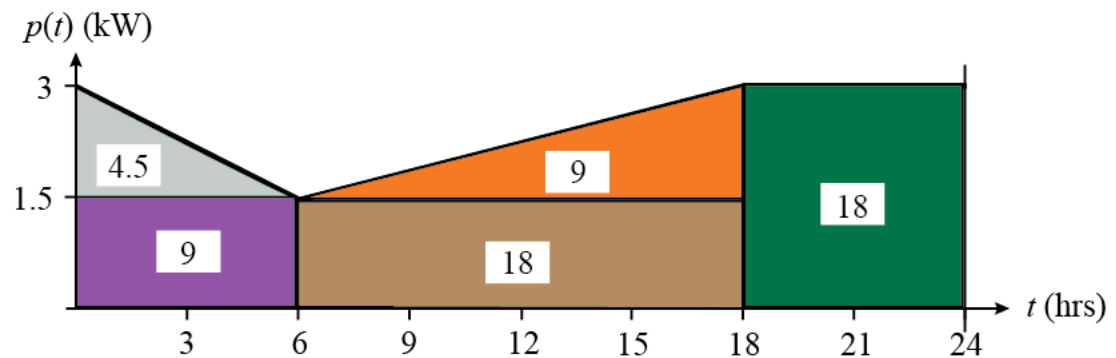


Note: above curve resembles actual data from the following source (8/28/2014):
<http://www.solarchoice.net.au/blog/how-do-i-use-electricity-throughout-the-day-the-load-curve/>

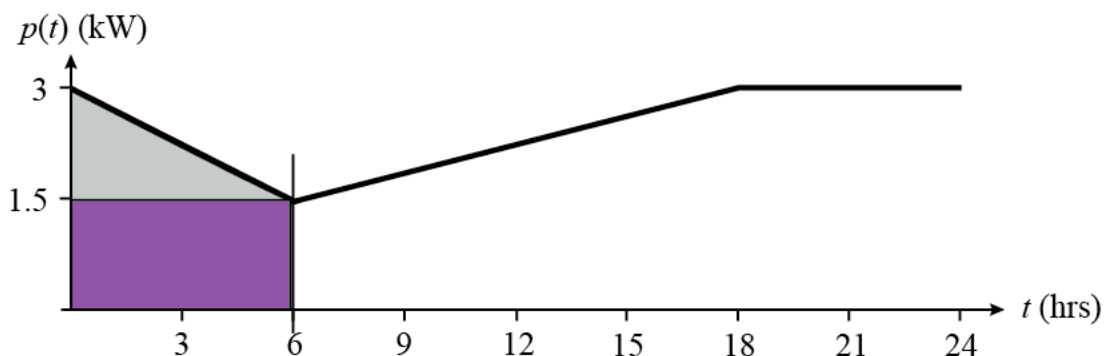
SOL'N: The final integral curve is shown below.



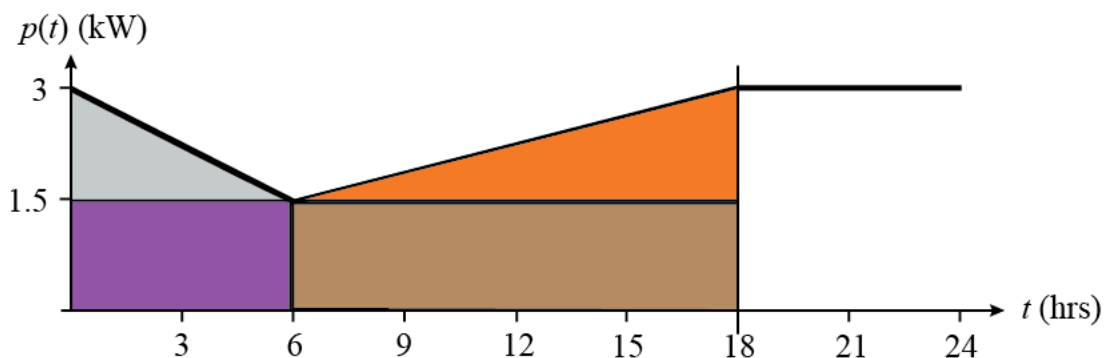
The curve was derived by finding the area under the curve between $t = 0$ and several key points in time shown by colored boxes on the above curve. For the key points, the area under the curve are simple shapes. The areas of the shapes are shown below:



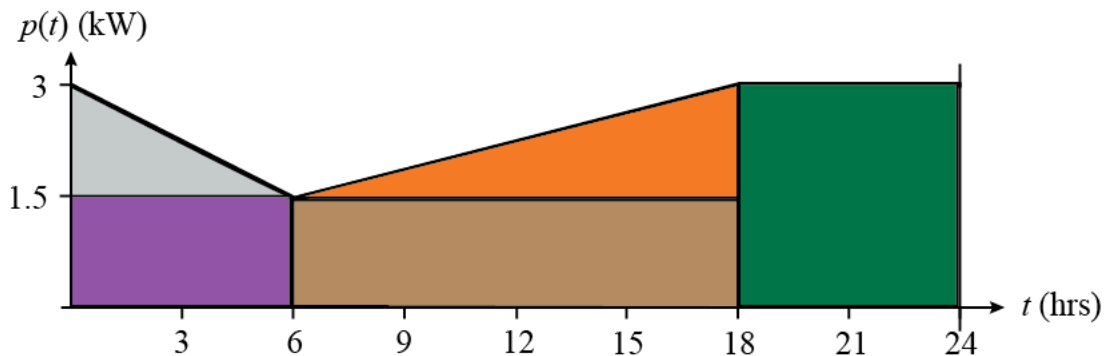
For the first key point at $t = 6$ ms, the area under the curve between $t = 0$ and 6 ms is 13.5 kWh:



For the second key point at $t = 18$ ms, the area under the curve between $t = 0$ and 18 ms is 40.5 kWh:

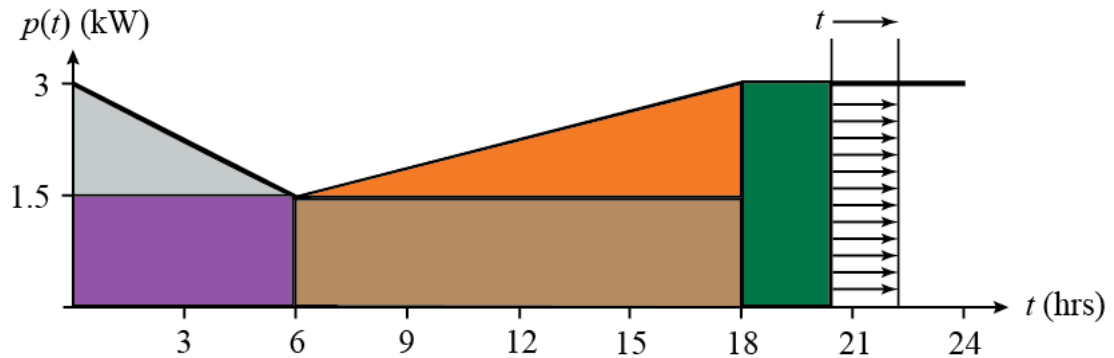


For the third key point at $t = 24$ ms, the area under the curve between $t = 0$ and 24 ms is 58.5 kWh:



Once the colored boxes are placed on the plot for the final answer, we can sketch in the portions in between by reasoning about the integrals of certain parts of the curve. For example, near the end of the curve, where the curve is flat, the area grows linearly as the time increases. The figure

below shows how the growing value of t yields a rectangle of growing width whose area increases linearly with the change in t . We could also see this mathematically: the integral of a constant function, k , is kt . That is, the integral is a line of slope k .



For the linearly-sloped regions, we are integrating lines, which gives quadratic functions. If the line slopes down, we get a concave quadratic function. If the line slopes up, we get a convex quadratic function.

The slope of the integral curve is the value of the $p(t)$ curve. Thus, the slope of the final answer at $t = 6$ ms is half the slope of the final answer at $t = 12$ ms. The slope of the final answer at $t = 24$ ms is the same as at $t = 18$ ms, since the power curve is constant between 18 ms and 24 ms.

Once again, the final curve:

