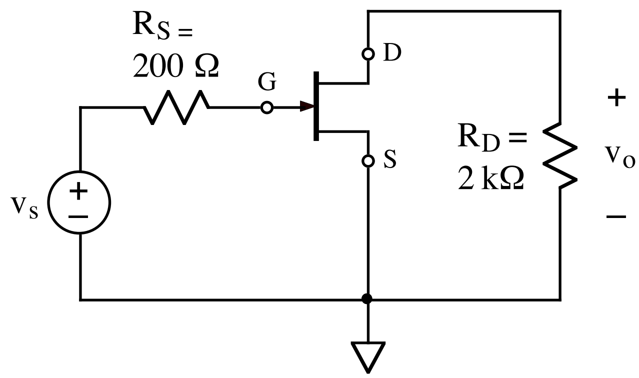
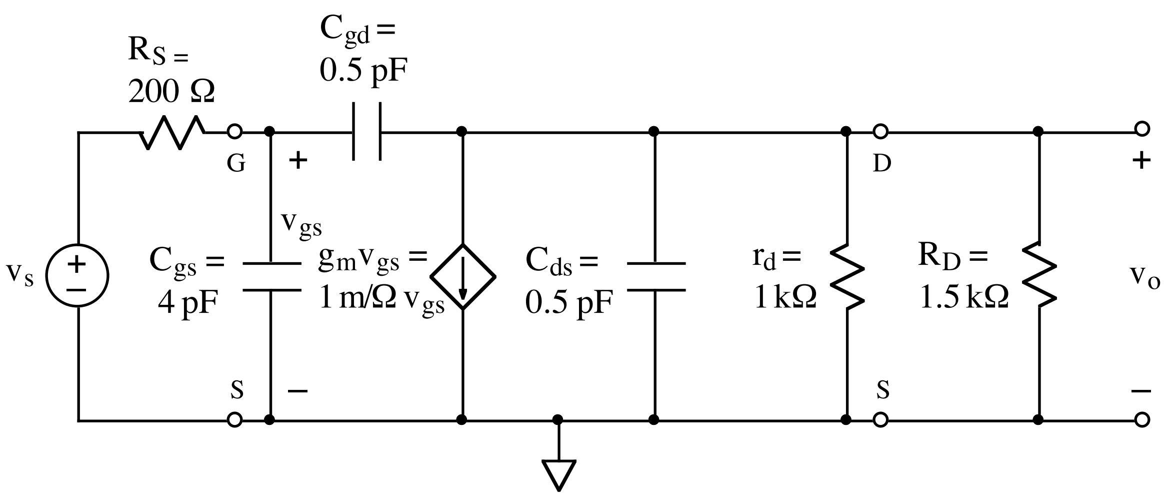
Ex:





$v_s(t) = 2 \cos(10\text{k}t)$ V

The above circuit diagrams show a common-source JFET amplifier and its high-frequency equivalent circuit. Find $v_o(t)$.

Sol'n: In this practical circuit, we have circuit values that allow us to make simplifying approximations.

We first calculate impedance values.

$\omega = 10\text{k r/s}$ from $v_s(t) = 2 \cos(10\text{k}t)$ V

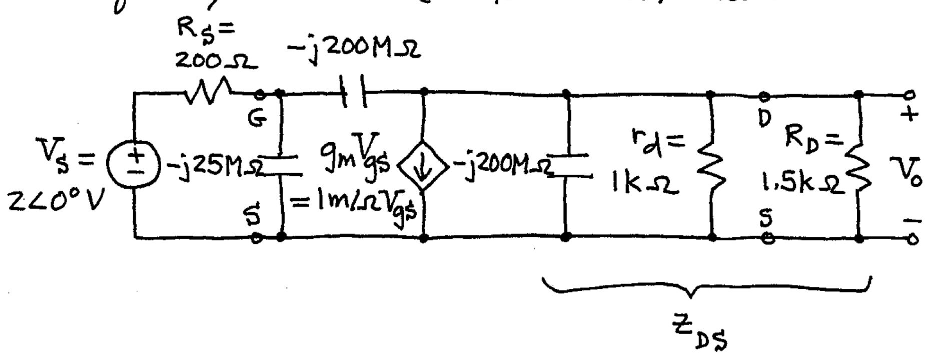
$$Z_{C_{gs}} = \frac{-j}{\omega C_{gs}} = \frac{-j\ \Omega}{10k \cdot 4p} = -j\,25M\,\Omega$$

$$Z_{C_{gd}} = \frac{-j}{\omega C_{gd}} = \frac{-j}{10k\,\frac{1}{2}p} = -j\,200M\,\Omega$$

$$Z_{C_{ds}} = \frac{-j}{\omega C_{ds}} = \frac{-j}{10k\,\frac{1}{2}p} = -j\,200M\,\Omega$$

The phasor for $v_s(t)$ is $V_s = 2\angle 0°$ V.

Frequency domain (or s-domain) model:



$$Z_{DS} = -j\,200M \,\|\, 1k\Omega \,\|\, 1.5k\Omega$$

Starting with $1k\Omega \,\|\, 1.5k\Omega$ we have

$$1k\Omega \,\|\, 1.5\,k\Omega = 500\Omega \cdot 2\|3 = 500 \cdot \frac{6}{5}$$

$$= 600\Omega$$

Thus, $Z_{DS} = -j\,200M \,\|\, 600\Omega = \dfrac{1}{\frac{1}{600} - \frac{1}{j200M}}$

Using $-\frac{1}{j} = j$ and rationalizing gives

$$z_{DS} = \frac{1}{\dfrac{1}{600} + \dfrac{j}{200M}} \cdot \frac{\dfrac{1}{600} - \dfrac{j}{200M}}{\dfrac{1}{600} - \dfrac{j}{200M}} \ \Omega$$

$$= \frac{\dfrac{1}{600} - \dfrac{j}{200M}}{\left(\dfrac{1}{600}\right)^2 + \left(\dfrac{1}{200M}\right)^2} \ \Omega$$

$$\approx \frac{\dfrac{1}{600} - \dfrac{j}{200M}}{\left(\dfrac{1}{600}\right)^2} \ \Omega \quad \text{Since } \frac{1}{200M} \ll \frac{1}{600}$$

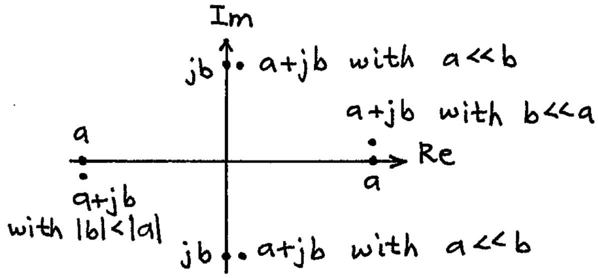
$$z_{DS} \approx \frac{\dfrac{1}{600}}{\left(\dfrac{1}{600}\right)^2} \ \Omega = 600\ \Omega$$

In retrospect, we could have made the approximation that $-j200M \| 600\ \Omega \approx 600\ \Omega$.

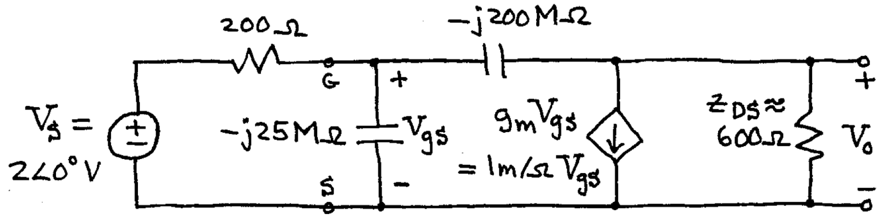
We may make this approximation despite the j in one of the quantities. In general, we may make the following approximations of complex values:

$$a + jb \approx a \quad \text{when } |b| \ll |a|$$

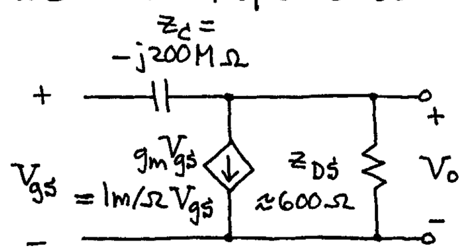
$$a + jb \approx jb \quad \text{when } |a| \ll |b|$$



With our z_{DS} value, we have a simplified model:



We now analyze the dependent source so we can replace it with an impedance, z_{eq}.



$$z_{eq} = \frac{V_o}{g_m V_{gs}}$$ using Ohm's law to write $z_{eq} = V/I$

Now we find a way to write V_o in terms of V_{gs}. We use a V-divider:

$$V_o = V_{gs} \frac{z_{eq} \| z_{DS}}{z_{eq} \| z_{DS} + z_C}$$

Substituting for V_o in our z_{eq} eq'n, we have

$$z_{eq} = \frac{\cancel{V_{gs}} \dfrac{z_{eq} \| z_{DS}}{z_{eq} \| z_{DS} + z_C}}{\cancel{V_{gs}} \cdot g_m}$$

$$z_{eq} = \frac{1}{g_m} \frac{\dfrac{z_{eq} z_{DS}}{z_{eq} + z_{DS}}}{\dfrac{z_{eq} z_{DS}}{z_{eq} + z_{DS}} + z_C}$$

$$z_{eq} = \frac{1}{g_m} \frac{z_{eq} z_{DS}}{z_{eq} z_{DS} + z_C (z_{eq} + z_{DS})}$$

Dividing top and bottom by z_{eq} gives the following:

$$z_{eq} = \frac{1}{g_m} \frac{z_{DS}}{z_{DS} + z_C + \frac{z_C z_{DS}}{z_{eq}}}$$

$$z_{eq}\left(z_{DS} + z_C + \frac{z_C z_{DS}}{z_{eq}}\right) = \frac{1}{g_m} z_{DS}$$

$$z_{eq}(z_{DS} + z_C) + z_C z_{DS} = \frac{1}{g_m} z_{DS}$$

$$z_{eq} \frac{(z_{DS} + z_C)}{z_{DS}} = \frac{\frac{1}{g_m} \cancel{z_{DS}} - z_C \cancel{z_{DS}}}{\cancel{z_{DS}}}$$

$$z_{eq} = \frac{\frac{1}{g_m} - z_C}{1 + \frac{z_C}{z_{DS}}} = \frac{\frac{1\Omega}{1m} - -j200\,M\Omega}{1 + \frac{-j200\,M\Omega}{600\Omega}}$$

$$z_{eq} = \frac{1k + j200M}{1 - j\frac{1}{3}M}\,\Omega$$

The imaginary parts of the numerator and denominator are much larger than the real parts. Thus, we ignore the real parts.

$$z_{eq} \approx j200M\Omega / -j\tfrac{1}{3}M \approx -600\,\Omega$$

Now we have a problem: $z_{eq} \| z_{DS} = -\dfrac{600^2}{0} \Omega$.

That means $z_{eq} \| z_{DS} = \infty\ \Omega$.

It is a good idea to try a more exact calculation to be sure that $z_{eq} \| z_{DS}$ is much larger than $z_C = -j200\,\text{M}\Omega$.

We use conductance to simplify calculations.

$$\frac{1}{z_{eq} \| z_{DS}} = \frac{1}{z_{eq}} + \frac{1}{z_{DS}} = \frac{1 + \dfrac{z_C}{z_{DS}}}{\dfrac{1}{g_m} - z_C} + \frac{1}{z_{DS}}$$

$$= \frac{1}{z_{DS}} \left(\frac{z_{DS} + z_C}{\dfrac{1}{g_m} - z_C} + 1 \right)$$

$$= \frac{1}{z_{DS}} \left(\frac{z_{DS} + z_C}{\dfrac{1}{g_m} - z_C} + \frac{\dfrac{1}{g_m} - z_C}{\dfrac{1}{g_m} - z_C} \right)$$

$$= \frac{1}{z_{DS}} \left(\frac{z_{DS} + \dfrac{1}{g_m}}{\dfrac{1}{g_m} - z_C} \right)$$

$$= \frac{g_m + \dfrac{1}{z_{DS}}}{1 - g_m z_C}$$

$$= \frac{1\text{m} + \dfrac{1}{600}}{1 - 1\text{m}(-j200\,\text{M})} \ /\Omega$$

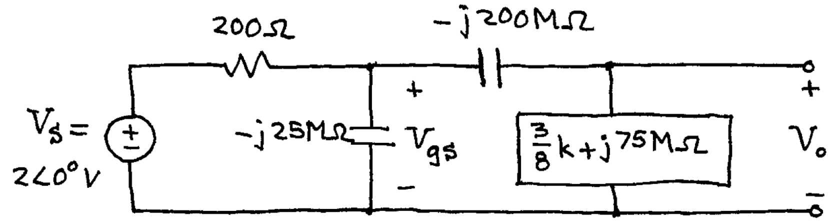
$$z_{eq} \| z_{DS} = \frac{1 + j200\,\text{k}}{\dfrac{1}{1\text{k}} + \dfrac{1}{600}} \ \Omega$$

$$z_{eq} \| z_{DS} = \frac{1 + j\,200\text{k}}{\frac{3+5}{3\text{k}}}\ \Omega = \frac{3}{8}\text{k}\,(1 + j200\text{k})\ \Omega$$

$$= \frac{3}{8}\text{k} + j\,75\text{M}\,\Omega$$

We see that the value is smaller than $z_C = -j\,200\ \text{M}\Omega$.

Our new, simplified model:



We use V-dividers to find V_o.

$$V_{gs} = V_s \cdot \frac{-j25\text{M}\Omega \,\|\, (-j200\text{M} + \overbrace{\tfrac{3}{8}\text{k}}^{\text{small}} + j\,75\text{M}\Omega)}{200 + -j25\text{M}\Omega \,\|\, (-j200\text{M} + \overbrace{\tfrac{3}{8}\text{k}}^{\text{small}} + j75\text{M}\Omega)}$$

$$V_{gs} \doteq \frac{V_s \cdot (-j25\text{M}\Omega \,\|\, -j\,125\text{M}\Omega)}{200 - j\,25\text{M}\Omega \,\|\, -j\,125\text{M}\Omega}$$

where $-j25\text{M}\Omega \,\|\, -j125\text{M}\Omega = -j25\text{M}\Omega \cdot 1\|5$

$$= -j25\text{M}\Omega \cdot \frac{5}{6}$$

$$= -j\,\frac{125}{6}\ \text{M}\Omega$$

$$V_{gs} = \frac{V_s \cdot \left(-j\,\frac{125}{6}\text{M}\Omega\right)}{\overbrace{200}^{\text{small}} - j\,\frac{125}{6}\text{M}\Omega} \approx V_s$$

$$V_o = V_{gs} \frac{\cancel{\tfrac{3}{8}k}^{\text{small}} + j75\,\text{M}\Omega}{\cancel{\tfrac{3}{8}k}^{\text{small}} + j75\,\text{M}\Omega - j200\,\text{M}\Omega}$$

$$V_o \approx V_{gs} \frac{j75\,\text{M}\Omega}{-j125\,\text{M}\Omega} = V_{gs}\left(-\frac{3}{5}\right)$$

$$V_o \approx 2\angle 0^\circ\,\text{V}\left(-\frac{3}{5}\right) = -\frac{6}{5}\angle 0^\circ\,\text{V} = \frac{6}{5}\angle 180^\circ\,\text{V}$$

Note: a minus sign is the same as 180° of phase shift.

$$v_o(t) = \frac{6}{5}\cos(10\text{k}t + 180^\circ)\ \text{V}$$