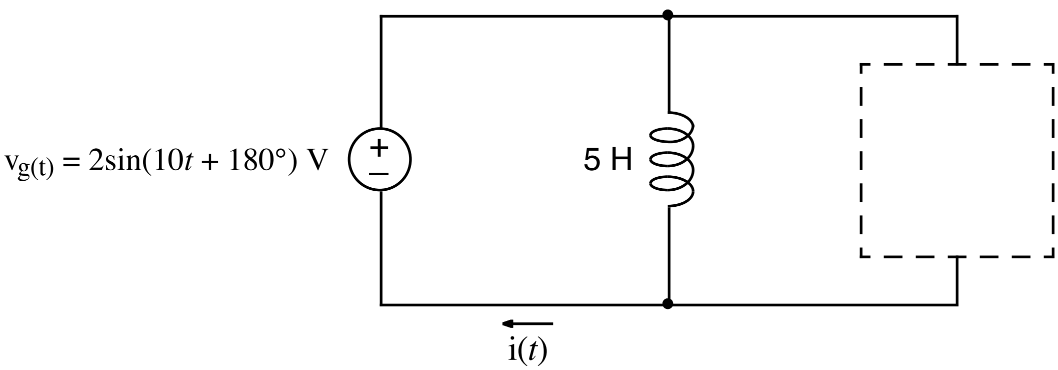
Ex:



a) Choose an R, an L, or a C to be placed in the dashed-line box to make

$$i(t) = I_o \cos(10t + 45°) \text{ A}$$

where $\mathbf{I_o}$ is a real constant. State the value of the component you choose.

b) With your component in the circuit, calculate the resulting value of $\mathbf{I_o}$.

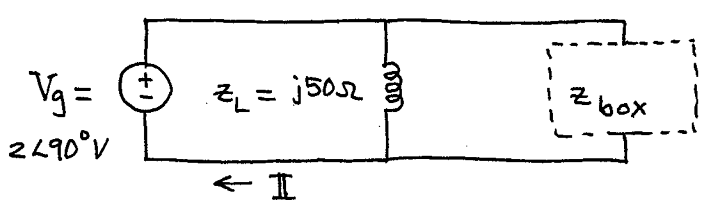
Sol'n: a) Transform to frequency domain.

$$V_g = -j2e^{j180°} \text{ V} \quad \text{since } P[\sin(\omega t)] = -j$$

$$V_g = +j2 \text{ V} = 2\angle 90° \text{ V}$$

$$Z_L = j\omega L = j \cdot 10 \text{ r/s} \cdot 5 \text{ H} = j50\,\Omega$$

Note: $\omega = 10$ r/s from $v_g(t)$.



We have $\mathbb{I} = \dfrac{V_g}{Z_L \| Z_{box}}$.

In terms of angles:

$\angle \mathbb{I} = 45°$ from $i(t) = I_0 \cos(10t + 45°)$ A

$$\angle \mathbb{I} = 45° = \angle \mathbb{V}_g - \angle(z_L \| z_{box})$$

or $$45° = 90° - \angle(z_L \| z_{box})$$

Thus, $\angle(z_L \| z_{box}) = 45°$.

For parallel components, it is easier to use conductance $= \frac{1}{z} \equiv g$

$$g_L = \frac{1}{z_L} \qquad g_{box} = \frac{1}{z_{box}}$$

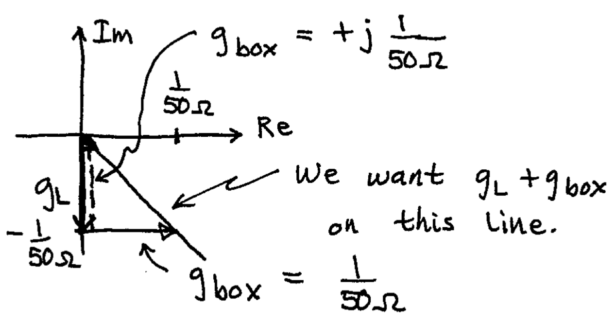
$$\angle(z_L \| z_{box}) = -\angle\left(\frac{1}{z_L \| z_{box}}\right) = -\angle\left(\frac{1}{z_L} + \frac{1}{z_{box}}\right)$$

or
$$\angle\left(\frac{1}{z_L} + \frac{1}{z_{box}}\right) = -\angle(z_L \| z_{box}) = -45°$$

or
$$\angle(g_L + g_{box}) = -45°$$

Now we use a plot:

$$g_L = \frac{1}{j50\Omega} = -j \cdot \frac{1}{50\Omega}$$



We have two possible solutions:

1) $g_{box} = j\frac{1}{50\Omega} \Rightarrow z_{box} = \frac{50\Omega}{j} = -j50\Omega$

Need capacitor: $z_C = \frac{-j}{\omega C} = \frac{-j}{10 \cdot C} = -j50\Omega$

$\Rightarrow C = \frac{1}{10 \cdot 50} F = 2\ mF$

or 2) $g_{box} = \frac{1}{50\Omega} \Rightarrow z_{box} = 50\Omega$ resistor

Either soln is technically correct, but solution (1) gives $g_L + g_{box} = 0 \Rightarrow z = \infty$.

Thus, no current flows for this soln.

b) Now use magnitude.

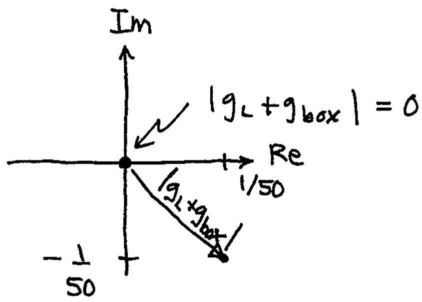
$$|I| = \left|\frac{V_g}{z_L \| z_{box}}\right| = \frac{|V_g|}{|z_L \| z_{box}|}$$

$\quad \|$

or $\quad I_0 = |V_g| \cdot |g_L + g_{box}|$

$$I_0 = 2 \cdot |g_L + g_{box}|$$

From diagram used to find $g_L + g_{box}$ on $-45°$ line, use magnitude of vector for $g_L + g_{box}$.



For sol'n (1), $|g_L + g_{box}| = 0 \quad \Rightarrow I_0 = 0\,A$

For sol'n (2), $|g_L + g_{box}| = \left| \frac{1}{50\Omega} - j\frac{1}{50\Omega} \right|$

$$= \left| \frac{1}{50\Omega}(1-j) \right|$$

$$= \frac{1}{50\Omega}\sqrt{1^2+1^2}$$

$$= \frac{\sqrt{2}}{50\Omega}$$

$$\Rightarrow I_0 = \frac{2\cdot\sqrt{2}}{50}A = \frac{\sqrt{2}}{25}\,A$$