TOOL: The steps presented below for the example circuit in Fig. 1 are an efficient way of computing the total equivalent of parallel impedances in general.

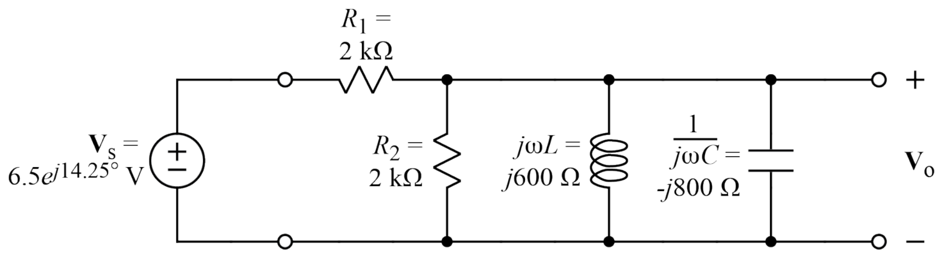

Fig. 1. Caption.

1) If the circuit is a voltage-divider, which many filters are, convert the voltage source and the resistor in the Thévenin configuration (V_s and R_1 in Fig. 1) to a Norton configuration, as in Fig. 2. In general, get all resistors in parallel.

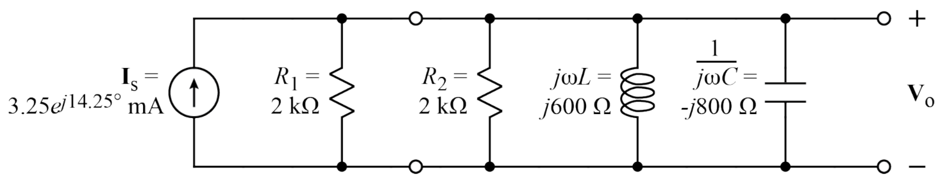

Fig. 2. V-source and resistor converted to Norton firm to create circuit model with all impedances in parallel.

2) Compute the equivalent of all the (real) parallel resistances. The idea is to group real impedances together and group imaginary impedances together to simplify calculations. Note that a set of parallel impedances may be rearranged to put all the resistances next to each other. That is, components in parallel may be shown in any order, so long as they are still in parallel.

   Remember to factor out an obvious common factors when computing parallel resistance. The parallel operator binds tighter than addition (is done first) in our calculations.

   For our example, we have

   $$R_1||R_2 = 2\,\text{k}\Omega||2\,\text{k}\Omega = 2\,\text{k}\Omega \cdot 1||1 = 2\,\text{k}\Omega \cdot \frac{1 \cdot 1}{1+1} = 1\,\text{k}\Omega.$$

3) Compute the equivalent of all the (imaginary) parallel resistances. Factor out j when calculating the equivalent impedance.

For our example, we have

$$j600\,\Omega || -j800\,\Omega = j200\,\Omega \cdot 3 || -4 = j2\,\Omega \cdot \frac{3 \cdot -4}{3 + -4} = j2.4\,\text{k}\Omega.$$

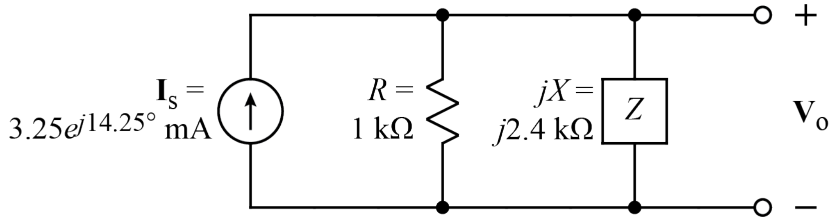

Fig. 3. Circuit reduced to three components in parallel.

4) Given the equivalent total resistance, R, and the equivalent total imaginary impedance, jX, use one of the following equivalent formulas for the equivalent impedance. (Remember that you can factor out a common factor before using these formulas.)

$$R || jX = \frac{RX}{R^2 + X^2}(X + jR) \qquad \text{Rectangular form} \qquad (1)$$

$$R || jX = \frac{RX}{\sqrt{R^2 + X^2}} e^{j\tan^{-1}(R/X)} \qquad \text{Polar form} \qquad (2)$$

For our example, we have the following calculations:

$$1\,\text{k}\Omega \cdot 1 || j2.4 = 1\,\text{k}\Omega \cdot \frac{2.4}{1^2 + 2.4^2}(2.4 + j) = \frac{2.4}{6.76}\text{k}\Omega(2.4 + j) \qquad \text{Rect}$$

$$1\,\text{k}\Omega \cdot 1 || j2.4 = 1\,\text{k}\Omega \cdot \frac{2.4}{\sqrt{1^2 + 2.4^2}} e^{j\tan^{-1}(1/2.4)} = \frac{2.4}{2.6} e^{j22.62°}\,\text{k}\Omega \qquad \text{Polar}$$

Note that the values are not in their most reduced form, which may be convenient in the next step.

We now have the equivalent circuit in Fig. 4.

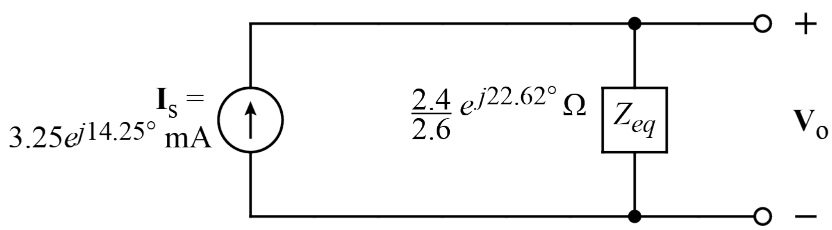

Fig. 4. Equivalent circuit model in reduced form.

5) The output voltage is now the product of the current source value and the total equivalent impedance. The polar form of the total equivalent impedance is convenient for our example.

$$\mathbf{V}_{\mathrm{o}} = 3.25\,\mathrm{mA}e^{j14.25^\circ} \cdot \frac{2.4}{2.6}e^{j22.62^\circ}\,\mathrm{k\Omega} = 3e^{j36.87^\circ}\,\mathrm{V}$$