

TOOL: Once a series impedance is simplified to a single (real) resistance, R , in series with a single imaginary impedance, jX , (1) gives the value of an equivalent single (real) resistance in parallel with a single imaginary impedance.

$$R + jX = \frac{R^2 + X^2}{R} \parallel j \frac{R^2 + X^2}{X} \quad \text{Rectangular form (1)}$$

$$|Z|e^{j\angle Z} = |Z|[\cos(\angle Z) + j\sin(\angle Z)] = \frac{|Z|}{\cos(\angle Z)} \parallel j \frac{|Z|}{\sin(\angle Z)} \quad \text{Polar form (2)}$$

VERIF: Verification of formulas (1) and (2).

$$\begin{aligned} \frac{R^2 + X^2}{R} \parallel j \frac{R^2 + X^2}{X} &= (R^2 + X^2) \cdot \frac{1}{R} \parallel j \frac{1}{X} \\ &= (R^2 + X^2) \cdot \frac{1}{R + \frac{X}{j}} \\ &= (R^2 + X^2) \cdot \frac{1}{R + \frac{X}{j}} \cdot \frac{R + jX}{R + jX} \\ &= (R^2 + X^2) \cdot \frac{R + jX}{R^2 + X^2} \\ &= R + jX \quad \text{Rect (1)} \end{aligned}$$

$$\begin{aligned} \frac{|Z|}{\cos(\angle Z)} \parallel j \frac{|Z|}{\sin(\angle Z)} &= |Z| \cdot \frac{1}{\cos(\angle Z)} \parallel j \frac{1}{\sin(\angle Z)} \\ &= |Z| \cdot \frac{1}{\cos(\angle Z) + \frac{\sin(\angle Z)}{j}} \\ &= |Z| \cdot \frac{1}{\cos(\angle Z) + \frac{\sin(\angle Z)}{j}} \cdot \frac{\cos(\angle Z) + j\sin(\angle Z)}{\cos(\angle Z) + j\sin(\angle Z)} \\ &= |Z| \cdot \frac{\cos(\angle Z) + j\sin(\angle Z)}{\cos^2(\angle Z) + j\sin^2(\angle Z)} \\ &= |Z|[\cos(\angle Z) + j\sin(\angle Z)] \quad \text{Polar (2)} \end{aligned}$$