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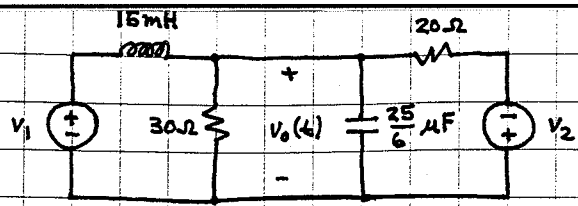
Ref: Nilsson & Riedel, Rev 6, Prob 9.32

ex:



$$v_1(t) = 240 \cos(4000t + 53.13°) \text{ V}$$
$$v_2(t) = 96 \sin(4000t) \text{ V}$$

Use source transformations to find steady-state $v_o(t)$.

soln: Find Norton equivalents for left and right sides. (Note: One could also use Thevenin equivalents.)
First, convert sources to phasors.

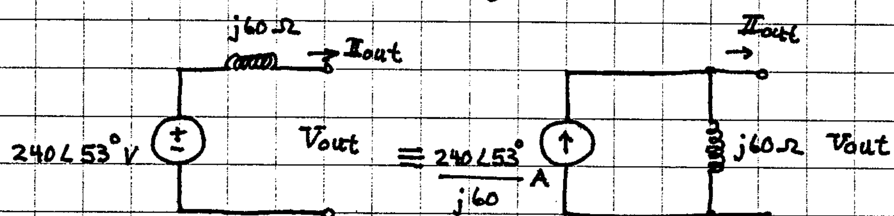
$$\mathbf{V}_1 = 240 \angle 53.13° \text{ V} \qquad \text{Note: } \omega = 4000 \text{ from } 4000t + \ldots$$

$$\mathbf{V}_2 = 96 \angle -90° \text{ V} \qquad \text{since } \sin(\omega t) = \cos(\omega t - 90°)$$

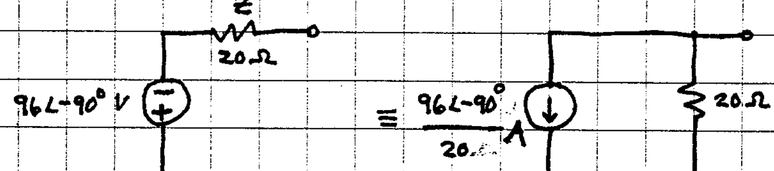
Second, convert L's and C's to impedances:

$$z_L = j\omega L = j\,4000 \cdot 15m = j\,60\ \Omega$$

$$z_C = \frac{-j}{\omega C} = \frac{-j}{4000 \cdot \frac{25}{6}\mu} = \frac{-j\,6}{100k \cdot \mu} = -j\,60\ \Omega$$

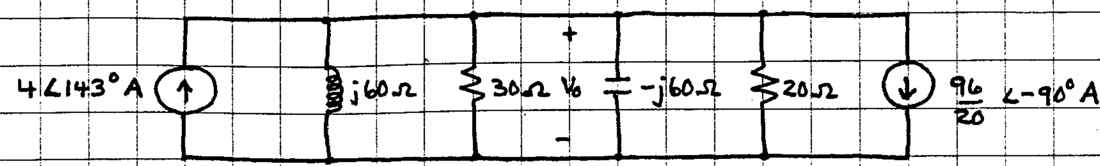


Make open-circuit $\mathbf{V}_{out}$'s match, and make short-circuit $\mathbf{I}_{out}$'s match. Result corresponds to case of DC v-source and R, but current source is now $\frac{\mathbf{V}}{z}$ and z replaces R across source.



New circuit with Norton equivalents:

$$240\angle 53^\circ / j60 = 240\angle 53^\circ / 60e^{+j90^\circ} = \frac{240}{60}\angle 53^\circ - 90^\circ$$



$$\text{Phasor } V_o = \left(4\angle -37^\circ - \frac{96}{20}\angle -90^\circ\right)\text{A} \cdot j60 \parallel 30 \parallel -j60 \parallel 20\ \Omega$$

$$4\angle -37^\circ = +3.19 - j2.41 \qquad \frac{96}{20}\angle -90^\circ = -j4.8$$

(Recall that $Re^{j\phi}$ or $R\angle\phi = R\cos\phi + jR\sin\phi$.)

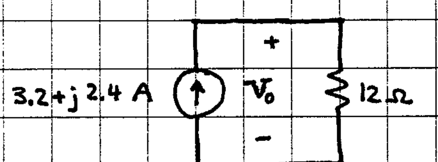
$$4\angle -37^\circ - \frac{96}{20}\angle -90^\circ = +3.19 - j2.41 - -j4.8 = 3.19 + j2.39$$

$$\approx 3.2 + j2.4$$

$$\text{Use} \quad \frac{1}{Z_{Tot}} = \cancel{\frac{1}{j60\Omega}} + \frac{1}{30\Omega} + \cancel{\frac{1}{-j60\Omega}} + \frac{1}{20\Omega}$$

$$\text{or} \quad Z_{Tot} = 30 \parallel 20\ \Omega = 10\Omega \cdot 3 \parallel 2 = 10\Omega \cdot \frac{3\cdot 2}{3+2} = 12\Omega$$

Simplified circuit:



$$V_o = (3.2 + j2.4\ \text{A}) \cdot 12\Omega$$
$$= 38.4 + j28.8\ \text{V}$$
$$= \left(\sqrt{3.2^2 + 2.4^2}\ \angle \tan^{-1}\frac{2.4}{3.2}\right)\cdot 12$$

$$V_o = 48\angle 37^\circ\ \text{V}$$

$$\therefore\ v_o(t) = 48\cos(4000t + 37^\circ)\ \text{V}$$