

THM: Given $f(t)$ and Laplace transform $F(s)$ with all poles in the left half-plane or at the origin as a single pole,

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s).$$

PROOF: The right side may be recognized as equivalent to the pole cover-up method of finding the coefficient of a single pole at $s = 0$ when expressing $F(s)$ as partial fractions.

$$F(s) = \frac{A}{s} + \dots$$

Note that, when there is a single pole at $s = 0$, the limit is unnecessary, and we may say

$$\lim_{t \rightarrow \infty} f(t) = sF(s) \Big|_{s=0}.$$

Note also that, for a system with all other poles in the left half-plane, a pole at $s = 0$ corresponds in the time domain to a constant value (or DC offset in circuit terms), which will be the final value, as the theorem suggests.

If there is no pole at $s = 0$, we are bound to get zero as our value on the right side. An answer of zero is correct if all other poles are in the left half-plane.

What happens if there are poles elsewhere? Here are the possibilities:

- 1) Poles in the right half-plane result in $f(t)$ growing without bound as $t \rightarrow \infty$. Thus, there is no final value in this case.
- 2) Conjugate poles on the imaginary axis result in $f(t)$ having terms of form $\cos(\omega t + \theta)$ that never settle to a final value. Thus, there is no final value in this case.
- 3) Two or more poles at the origin result in $f(t)$ having terms of form t^n for $n > 1$ that grow without bound as $t \rightarrow \infty$. Thus, there is no final value in this case.

From the above discussion, the Final Value Theorem is useful only when $F(s)$ has a single pole at the origin and all other poles in the left half-plane. In that case, use pole cover-up to find the partial-fraction coefficient for the $1/s$ term.