

THM: Given $f(t)$ and Laplace transform $F(s)$,

$$\lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} sF(s)$$

PROOF: Using the definition of the Laplace transform, expand the expression on the right side.

$$\lim_{s \rightarrow \infty} sF(s) \equiv \lim_{s \rightarrow \infty} s \int_{0^-}^{\infty} f(t) e^{-st} dt$$

Since integration is a linear operator and since s is not the variable of integration, move the limit inside the integral.

$$\lim_{s \rightarrow \infty} sF(s) = \int_{0^-}^{\infty} f(t) \lim_{s \rightarrow \infty} s e^{-st} dt$$

The area of $s e^{-st}$ is equal to one for all values of s , and this function converges to an impulse function as s goes to infinity.

$$\lim_{s \rightarrow \infty} s e^{-st} dt = \delta(t)$$

Our expression is now reduced to an integral of our function times an impulse.

$$\lim_{s \rightarrow \infty} sF(s) = \int_{0^-}^{\infty} f(t) \delta(t) dt.$$

The integral of an impulse times a function just picks out the value of the function at the time the impulse occurs. Since e^{-st} converges to an impulse by narrowing on the right side, our integral picks up any initial conditions such as those created by impulses at $t = 0$. So we get the value of $f(t)$ as t approaches zero from the right.

$$\lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} sF(s)$$