Neil E Cotter Linear Algebra — 2x2 matrix
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$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det A = ad - bc$$

$$\det A = 0 \Rightarrow A \text{ is singular, noninvertible}$$

If A singular then 2nd row is multiple of 1st row
and 2nd col " " " 1st col.

ex: $A = \begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix}$ $\det A = -1\cdot 6 - (-2)3 = 0$

2nd row $= 3 \cdot$ 1st row
2nd col $= -2 \cdot$ 1st col

ex: $A = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}$ $\det A = 0\cdot 2 - 0\cdot 1 = 0$

$0 \cdot$ 2nd row $=$ 1st row
2nd col $= 2 \cdot$ 1st col

$$\det(A - \lambda I) = \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = (a-\lambda)(d-\lambda) - bc$$

eigenvalues: $\lambda^2 - (a+d)\lambda + ad - bc = 0$

or $\lambda^2 - (\text{Trace } A)\lambda + \det A = 0$

where $\text{Trace } A \equiv \text{Tr } A =$ sum of diagonal entries of A

$$\lambda = \frac{\text{Tr } A}{2} \pm \sqrt{\left(\frac{\text{Tr } A}{2}\right)^2 - \det A}$$

λ's *imaginary if and only if* $\det A > 0$ and $\det A > \left(\frac{\text{Tr } A}{2}\right)^2$