24 April 1988 Approximation Theory — Universal Approximation —

Neil E Cotter

claim: Perceptron is Turing equivalent

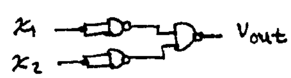
- A computational machine is Turing equivalent if it can compute anything that can be computed by a Turing machine — a simple computing machine (the Turing machine) that can compute anything so long as it has enough memory.

- Hence, the idea is that a Turing equivalent machine can compute anything another computer can compute.

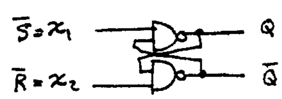
pf: Make Perceptron = NAND gate 

- A computer can be constructed entirely from NAND gates.

ex: NOT gate 

OR gate 

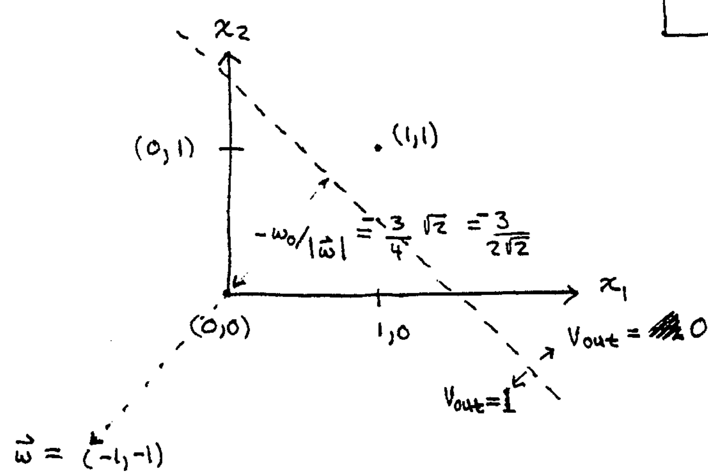
- 3 perceptrons used

RS Flip-Flop 

NAND gate

x_1	x_2	V_{out}
0	0	1
0	1	1
1	0	1
1	1	0

- Note: Our Perceptron NAND gate will also give binary V_{out} when x_1, x_2 are not binary, e.g. $(x_1, x_2) = (\frac{1}{2}, \frac{1}{2})$ might give $V_{out} = 1$.



$\vec{w} = (-1,-1)$

$|\vec{w}| = \sqrt{2}$

$w_0 = \frac{3}{2}$

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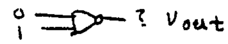
Approximation Theory – Universal Approximation – Perceptron is Turing Equivalent (cont.)

check: $(x_1, x_2) = (0,0)$

$$u = 1 \cdot w_0 + x_1 w_1 + x_2 w_2 = 1 \cdot \frac{3}{2} + 0 \cdot (-1) + 0(-1) = \frac{3}{2} > 0$$

$\therefore V_{out} \equiv g(u) = 1$ since $u > 0$ ✓

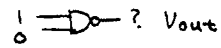
$(x_1, x_2) = (0,1)$



$$u = 1 \cdot \frac{3}{2} + 0 \cdot (-1) + 1(-1) = \frac{3}{2} - 1 = \frac{1}{2} > 0$$

$\therefore V_{out} \equiv g(u) = 1$ since $u > 0$ ✓

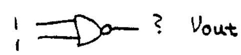
$(x_1, x_2) = (1,0)$



$$u = 1 \cdot \frac{3}{2} + 1 \cdot (-1) + 0 \cdot (-1) = \frac{3}{2} - 1 = \frac{1}{2} > 0$$

$\therefore V_{out} = 1$ ✓

$(x_1, x_2) = (1,1)$



$$u = 1 \cdot \frac{3}{2} + 1(-1) + 1(-1) = \frac{3}{2} - 1 - 1 = -\frac{1}{2} < 0$$

$\therefore V_{out} = 0$ since $u < 0$ ✓