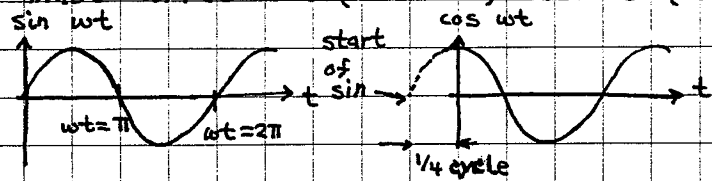
ex: a) If $f = 2\sin(\omega t + \pi/3)$ find $\mathcal{P}\{f\}$, (i.e. phasor).
(cont.)

soln: Draw picture of magnitude and phase at time $t=0$ such that real part $= 2\sin(\omega t+\pi/3)\big|_{t=0}$

$R = 2$ is magnitude

must write $\sin(\omega t + \pi/3)$ as $\cos(\omega t + \phi)$



Note that if we subtract $\frac{1}{4}$ cycle from ωt, then cos becomes sin: $\cos\left(\omega t - \frac{\pi}{2}\right) = \sin(\omega t)$

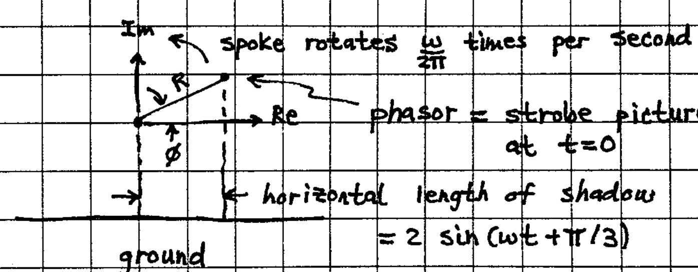
Thus, $\sin\left(\omega t + \frac{\pi}{3}\right) = \cos\left(\omega t - \frac{\pi}{2} + \frac{\pi}{3}\right) = \cos\left(\omega t - \frac{\pi}{6}\right)$

$\therefore\ f = 2\cos\left(\omega t - \frac{\pi}{6}\right)$

$\mathcal{P}\{f\} = Re^{j\phi} \qquad R = 2,\ \phi = -\frac{\pi}{6}$

$\mathcal{P}\{f\} = 2e^{-j\frac{\pi}{6}}$ or $2\angle -30^\circ$

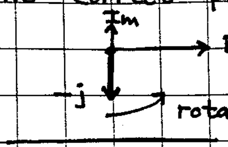
alternate soln: Think of phasor as strobed picture of spoke rotating around origin. The length of the horizontal shadow of the spoke over time is equal to $f = 2\sin(\omega t + \pi/3)$.



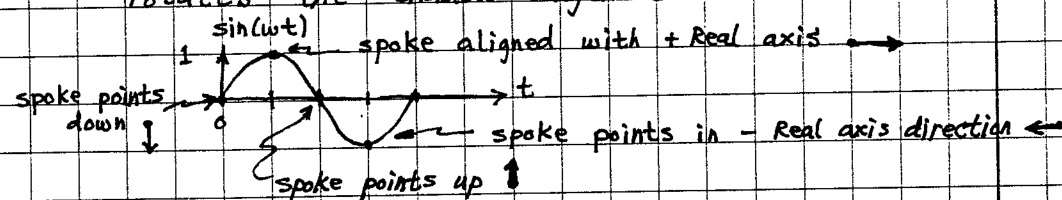
Now to find correct R and ϕ.

First, consider phasor for $\sin(\omega t)$. The shadow of the spoke should start at 0 for $t=0$: $\sin(0) = 0$. $\frac{1}{4}$ cycle later, the shadow length should be 1, since $\sin(\pi/2) = 1$

a) (cont) The correct phasor for $\sin(\omega t)$ is $-j$:



Note that at $t=0$ the spoke points straight down and the shadow length is zero. As the spoke rotates the shadow length traces out $\sin(\omega t)$:



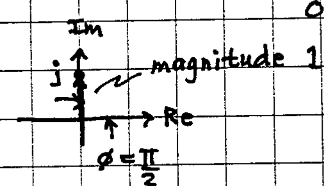
The phasor for $2\sin(\omega t + \frac{\pi}{3})$ will be the phasor for $\sin(\omega t)$ multiplied by a phase shift term $e^{j\pi/3}$ and a magnitude 2:

$$\mathcal{P}\{f\} = -2j\, e^{j\pi/3}$$

This looks different from our previous answer $2e^{-j\frac{\pi}{6}}$ but it is the same complex number:

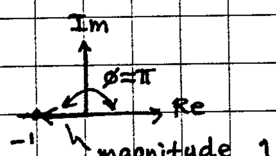
$$j = e^{j\pi/2}$$

This identity follows from writing $0 + j1$ in polar form as $1 \cdot e^{j\pi/2}$.



$$-1 = e^{\pm j\pi}$$

Again this follows from writing $-1 + j0$ in polar form as $1 \cdot e^{j\pi}$. We can use $e^{j\pi}$ or $e^{-j\pi}$, whichever is more convenient in a given situation.



Thus, $-2je^{j\pi/3} = e^{-j\pi} \cdot 2 \cdot e^{j\frac{\pi}{2}} e^{j\frac{\pi}{3}} = 2e^{j\left(\frac{\pi}{2} + \frac{\pi}{3} - \pi\right)}$

$$= 2e^{-j\frac{\pi}{6}} \text{ or } 2\angle -\frac{\pi}{6} \text{ or } 2\angle -30^\circ$$