TOOL: By shifting time t, the phase shift of an AC current waveform may be set to zero, and the phase shift of the AC voltage waveform may be written as the difference, ϕ_{v-i}, between the voltage phase shift and the current phase shift:

$$i(t) = I_m \cos(\omega t)\text{, and}$$

$$v(t) = V_m \cos(\omega t + \phi_{v-i}).$$

The power waveform may then be written as follows:

$$p(t) = \mathbf{P} + \mathbf{P}\cos(2\omega t) - \mathbf{Q}\sin(2\omega t)$$

where

$$\mathbf{P} = \frac{V_m I_m}{2}\cos(\phi_{v-i}) \text{ and}$$

$$\mathbf{Q} = \frac{V_m I_m}{2}\sin(\phi_{v-i})$$

TOOL: The phasor for the AC power waveform, is known as complex power, **S**, and while **S** technically describes only the sinusoidal part of the waveform, the real part of **S** is equal to the Average Power (a.k.a. DC power), **P**:

$$\mathbf{S} = \mathbf{P} + j\mathbf{Q}$$

Thus, the complex power (phasor) contains all the information about the power.

DEF: **P** ≡ Average Power ≡ DC power ≡ Active power ≡ Real power, units = Watts (W)

DEF: **Q** ≡ Reactive Power, units volt-amps reactive = (VAR)

DEF: **S** ≡ Complex Power, units volt-amps = (VA)

DEF: |**S**| ≡ Apparent Power = $\sqrt{\mathbf{P}^2 + \mathbf{Q}^2}$, units = (VA)

DEF: Power Factor ≡ $\cos(\theta_{v-i})$ = **P**/**S**, unitless

NOTE: All power units are equivalent to Watts, but distinct letters are used so that power terms for **P**, **Q**, and **S**, may be distinguished from one another by the units when they are written as numbers. (Note that **S** may lack an imaginary part.)

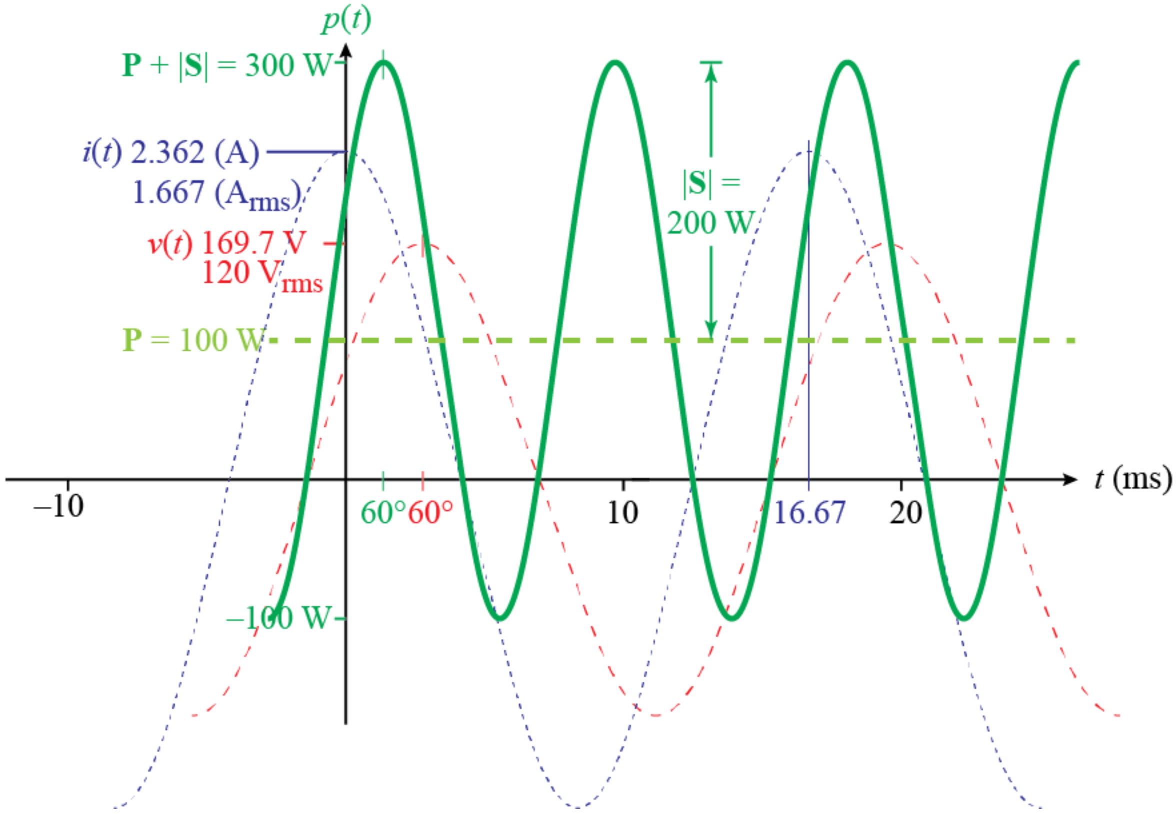
TOOL: Complex power, **S**, may be computed using the following formulas involving phasors **I** and **V** for current and voltage, and/or impedance Z:

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{|\mathbf{I}|^2 Z}{2} = \frac{|\mathbf{V}|^2}{2Z^*} = |\mathbf{I}_{\text{rms}}|^2 Z = \frac{|\mathbf{V}_{\text{rms}}|^2}{Z^*}$$

where $*$ ≡ conjugate (change all j's to $-j$'s), $\mathbf{I} = I_m \angle 0°$, $\mathbf{V} = V_m \angle \phi_{v-i}$,

and rms ≡ root-mean-square = $\sqrt{\frac{1}{T}\int_{t=0}^{T} f^2(t)\,dt}$ where T is one period of waveform.

TOOL: The figure below shows the AC waveforms for v, i, and p with variables related to complex power indicated. 100 W average power, 120 VAC at 60 Hz.



time-domain waveform	phasor
$i(t) = 2.362\cos(\omega t)\,\text{A}$	$\mathbf{I} = 2.362\angle 0°\,\text{A}$
$v(t) = 169.7\cos(\omega t - 60°)\,\text{V}$	$\mathbf{V} = 169.7\angle -60°\,\text{V}$

$$p(t) = \mathbf{P} + \mathbf{P}\cos(2\omega t) - \mathbf{Q}\sin(2\omega t) \qquad \mathbf{S} = \mathbf{P} + j\mathbf{Q} = \frac{|\mathbf{V}||\mathbf{I}|}{2}\cos(-60°)\,\text{W} + j\frac{|\mathbf{V}||\mathbf{I}|}{2}\sin(-60°)\,\text{VAR}$$