Ex: Draw all possible representative Venn diagrams consistent with the following information about events $A, B$, and $C$ : (use areas approximating probabilities)

$$
\begin{array}{lll}
\mathrm{P}(A)=0.225 & \mathrm{P}(B)=0.4 & \mathrm{P}(C)=0.9 \\
\mathrm{P}\left(A \cap B^{\prime}\right)=0.125 & & \mathrm{P}(B \cap C)=0.4
\end{array}
$$

SOL'N: We start our Venn diagram by drawing the most probable event, $C$ :


Total area of the diagram $=$ total probability of sample space $=1$. The area for event $\mathrm{C}=\mathrm{P}(\mathrm{C})=0.9$.

We observe that $\mathrm{P}(\mathrm{B})=\mathrm{P}(\mathrm{B} \cap \mathrm{C})=0.4$. This implies that event B lies entirely inside event $C$. (Note: strictly speaking, event $B$ might contain outcomes of zero probability that lie outside C . Thus, we are ignoring such outcomes. This is not a serious problem since the probability of these outcomes is zero.)


Given $\mathrm{P}(A)=0.225$ and $\mathrm{P}\left(A \cap B^{\prime}\right)=0.125$, we apply the law of total probability that says $\mathrm{P}(A \cap B)+\mathrm{P}\left(A \cap B^{\prime}\right)=\mathrm{P}(A)$ to conclude that $\mathrm{P}(A \cap B)=0.1$.

What we still lack is information about the intersection of $A$ and $C$. It is possible that $A$ lies entirely in $C$. So one possible representative diagram is as follows:


We know that $A$ intersects $B$, but might the rest of $A$ lie outside $C$ ? Or does $A$ have to intersect the part of $C$ outside of $B$ ? The probability of being in the part of A lying outside of B is $\mathrm{P}\left(A \cap B^{\prime}\right)=0.125$. But $\mathrm{P}\left(\mathrm{C}^{\prime}\right)=1-\mathrm{P}(\mathrm{C})=0.1$. Thus, there is too little room outside of C to fit all of A not lying in B . The minimum value for $\mathrm{P}\left(A \cap B^{\prime}\right)$ is 0.025 as shown in the other representative diagram:


