Ex: A sharp-toothed manager at NASA suspects that more parts are failing on space probes when they are installed while the moon is waxing. He has noticed several employees trembling from drinking too much coffee, and he has had to reprimand several employees for howling. But he wants to prove his theory. He has calculated the following quantities: $(\mathrm{F}=$ failed part, $\mathrm{W}=$ waxing moon $)$

$$
\mathrm{P}(\mathrm{~F} \mid \mathrm{W})=1 / 5 \quad \mathrm{P}(\mathrm{~W})=1 / 2 \quad \mathrm{P}(\mathrm{~W} \mid \mathrm{F})=2 / 3
$$

Sunlight is painful to the manager, and he wants to calculate $\mathrm{P}(\mathrm{F})$ before morning without gathering more data. Find $\mathrm{P}(\mathrm{F})$ for him.

SOL'N: Use the basic equation for conditional probability:

$$
\mathrm{P}(\mathrm{~F} \mid \mathrm{W}) \mathrm{P}(\mathrm{~W})=\mathrm{P}(\mathrm{~F}, \mathrm{~W})=\mathrm{P}(\mathrm{~W} \mid \mathrm{F}) \mathrm{P}(\mathrm{~F})
$$

Rearranging gives

$$
\mathrm{P}(\mathrm{~F})=\mathrm{P}(\mathrm{~F} \mid \mathrm{W}) \mathrm{P}(\mathrm{~W}) / \mathrm{P}(\mathrm{~W} \mid \mathrm{F})=1 / 5 \cdot 1 / 2 / 2 / 3=3 / 20=0.15 .
$$

