Conditional Probability

math: \( P( A \mid B) = \frac{\text{Probability of event } A \text{ given event } B \text{ occurred}}{B = \{2, 4, 6\}} \)

ex: Loaded die

\[ A = \{1, 2, 3, 5, 6\} \]

\[ B = \{2, 4, 6\} \]

- What is \( P( A \mid B) \) given that we know \( B \) occurred?
- Sample space \( \mathcal{S} \) becomes \( B \)
- Scale probs in \( B \) so they add up to one
  - Divide original probs by \( P( B) = \sum \text{ probs in } B \)
  - here \( P( B) = P(2) + P(4) + P(6) = \frac{2}{21} + \frac{2}{21} + \frac{4}{21} = \frac{4}{7} \)

\[ \therefore P(2 \mid B) = \frac{P(2)}{P( B)} = \frac{2}{4/7} = \frac{7}{4} = \frac{21}{4} \]

\[ P(4 \mid B) = \frac{P(4)}{4/7} = \frac{4}{4} = \frac{21}{3} \]

\[ P(6 \mid B) = \frac{P(6)}{4/7} = \frac{6}{4} = \frac{21}{2} \]

- \( P( A \mid B) = \frac{\text{events in } A \text{ that are in } B = A \cap B}{P( B) = 4/7} \)

\[ P( A \mid B) = \frac{P(2 \cap B) + P(6 \cap B)}{4/7} \]

\[ = \frac{P(2) + P(6)}{4/7} = \frac{2}{21} + \frac{4}{21} = \frac{3}{7} \]

- \( P( A \mid B) = \frac{P( A \cap B)}{P( B)} \)

**Tool:** \( P( A \mid B) = \frac{P( A \cap B)}{P( B)} \)

**ex:** \( P( A \mid B) = \frac{P( A \cap B)}{P( B)} \)

\( \therefore P( A \mid B) = \frac{P( A \cap B)}{P( B)} = 1 - P( B) \)

**ex:** \( P( A \mid B) = \frac{P( A \cap B)}{P( B)} = \frac{P( A) - P( A \cap B)}{1 - P( B)} = \) no simple relationship
**tool:** Multiplication law

\[ P(E \cap \bar{B}) = P(E)P(\bar{B}) \quad \text{very useful} \]

by:

\[ P(E \cap B \cap C) = P(E \cap B)P(C) \]
\[ = P(E)P(B \cap C)P(C) \]
\[ = P(E)P(C \cap B) \]
\[ = P(E)P(A \cap B)P(E) \]

**recursive**

**def:** Independent Events A and B \( \iff P(E \cap B) = P(E)P(B) \)

**note:** If \( P(E \cap B) = P(E)P(B) \)

then \( P(E \cap B) = P(E) \), i.e.

\[ P(E \cup B) = P(E) \]

and

\[ P(E \cap B) = P(E)P(B) \]

**ex:** independent events A, B, and C \( \Rightarrow P(E \cap A \cap B \cap C) = P(A)P(B)P(C) \)

**comments:** Independent \( \Rightarrow \) event gives no information about other event
Independent \( \Rightarrow \) no physical interaction between events (sort of)
Independent \( \Rightarrow \) can treat each event as being in separate axis

**ex:** Two dice red & green. Number on red die is independent of number on green die.

A\# Green Die: 6\# Doubles

B\# Red Die: 1\# & 6\# Doubles, 2\# & 5\# Doubles, 3\# & 4\# Doubles, 7\# Lucky 7

\[ \Rightarrow \# \text{on red die} \]