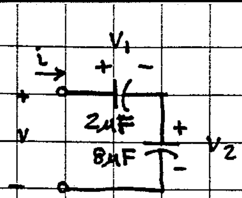
ex:



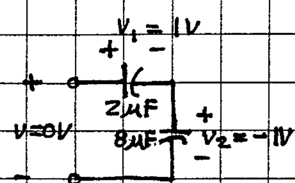
$i = 240e^{-10t}$ µA $\quad t \geq 0$

$v_1(t=0) = -10V \quad v_2(t=0) = -5V$

Calculate total energy trapped in C's as $t \to \infty$.
Hint: Don't combine C's in series — find energy for each C and sum them.

Note: The energy trapped in C's refers to energy that we cannot extract from the C's when the total V across the two C's in series is 0V.

For example:



Here we have $v_1 + v_2 = v = 0V$, but each C has $v_{1 or 2} \neq 0$ across it.

If we connect an R across the terminals, we will get no current flow because $v = 0V$. Thus, we cannot access the "trapped" energy.

The trapped energy is $w_{C1} + w_{C2} = \frac{1}{2} 2\mu F(+1V)^2 + \frac{1}{2} 8\mu F(-1V)^2$
$= 1\mu J + 4\mu J = 5\mu J.$

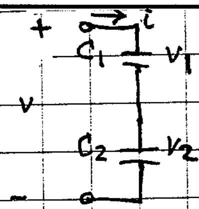
(Note that for series combination we would get $\frac{1}{2} C_{eq} v^2 = \frac{1}{2} C_{eq} \cdot 0V^2 = 0J$.)

Note: The energy stored in C's refers to energy that we can extract from the C's by connecting a circuit to the two terminals having series C's between.

For the stored energy, we get the correct answer if we use $C_{eq} = C_1 \| C_2 = \dfrac{C_1 C_2}{C_1 + C_2}$, (see next page).

moral: We can use C_{eq} if all we care about are the i and v at terminals, (or the energy we can extract from the terminals).

We now consider this energy issue in detail before finally solving the problem.



Assume no energy is trapped when $v=0$.
Now let current i flow until time t.

$i = C\dfrac{dv}{dt} \Rightarrow \int i\,dt = C_i v_i$ for each $C_i = C_1$ or C_2.

Both C's see same i (since in series), so $\int i\,dt$ same for both C_1 and C_2.

$$\therefore\ C_1 v_1 = C_2 v_2 \quad \text{or} \quad v_2 = \frac{C_1}{C_2} v_1$$

Now, $C_{eq} = C_1 \| C_2 = \dfrac{C_1 C_2}{C_1 + C_2}$.

$$\frac{1}{2} C_{eq} v^2 = \frac{1}{2} C_{eq} (v_1 + v_2)^2 = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} \left(v_1 + \frac{C_1}{C_2} v_1\right)^2$$

$$= \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} \left[\frac{(C_1 + C_2) v_1}{C_2}\right]^2$$

$$= \frac{1}{2} \frac{C_1}{C_2} (C_1 + C_2) v_1^2$$

$$\frac{1}{2} C_1 v_1^2 + \frac{1}{2} C_2 v_2^2 = \frac{1}{2} C_1 v_1^2 + \frac{1}{2} C_2 \left(\frac{C_1}{C_2} v_1\right)^2$$

$$= \frac{1}{2}\left(C_1 + \frac{C_1^2}{C_2}\right) v_1^2$$

$$= \frac{1}{2} \frac{C_1}{C_2} (C_1 + C_2) v_1^2$$

$$= \frac{1}{2} C_{eq} v^2 \quad \checkmark$$

If there is trapped energy, then when $v=0$ we have $v_1 = v_0$ and $v_2 = -v_0$ for some voltage v_0.
If we let current i flow and store energy then we have

$$\int i\,dt = C_1 (v_1 - v_0) = C_2 (v_2 + v_0), \quad \text{or} \quad v_2 = \underbrace{\frac{1}{C_2}\left[C_1 v_1 - (C_1 + C_2) v_0\right]}$$

The stored energy on C_{eq} is $\frac{1}{2} C_{eq} v^2 = \frac{1}{2} C_{eq} (v_1 + v_2)^2$.

This is equal to the total energy for C_1 and C_2 minus the trapped energy. In other words the stored energy is

$$\underbrace{\frac{1}{2} C_1 v_1^2 + \frac{1}{2} C_2 v_2^2}_{\text{total energy}} - \underbrace{\left(\frac{1}{2} C_1 v_0^2 + \frac{1}{2} C_2 v_0^2\right)}_{\text{trapped energy}} = \underbrace{\frac{1}{2} C_{eq} v^2}_{\text{stored energy}}$$

We now find $v_1(t\to\infty)$ and $v_2(t\to\infty)$.

$$i = 240e^{-10t}\ \mu A = C_1\frac{dv_1}{dt} = C_2\frac{dv_2}{dt}$$ same i flows thru capacitors in series

$$\int_{t=0}^{t=\infty} i(t)\,dt = \int_{t=0}^{t=\infty} 240e^{-10t}\,\mu A\,dt = \int_{v_1(t=0)}^{v_1(t=\infty)} C_1\,dv_1 = \int_{v_2(t=0)}^{v_2(t=\infty)} C_2\,dv_2$$

$$\frac{240}{-10}e^{-10t}\,\mu A s\Big|_{t=0}^{t=\infty} = C_1\left[v_1(t=\infty)-v_1(t=0)\right] = C_2\left[v_2(t=\infty)-v_2(t=0)\right]$$

$$-24\underbrace{e^{-\infty}}_{0} - -24\underbrace{e^{-0}}_{1}\ \mu A\cdot s = 2\mu F\left[v_1(t=\infty) - -10V\right] = 8\mu F\left[v_2(t=\infty) - -5V\right]$$

$$24\mu A s = 2\mu F\left[v_1(t=\infty)+10V\right] = 8\mu F\left[v_2(t=\infty)+5V\right]$$

$$v_1(t=\infty) = \frac{24\mu A s}{2\mu F} - 10V = 12V - 10V = 2V$$

$$v_2(t=\infty) = \frac{24\mu A s}{8\mu F} - 5V = 3V - 5V = -2V$$

$$\text{Energy trapped} \equiv w_{trapped} = \frac{1}{2}C_1v_1^2(t=\infty) + \frac{1}{2}C_2v_2^2(t=\infty)$$

$$= \frac{1}{2}2\mu F\cdot(2V)^2 + \frac{1}{2}8\mu F(-2V)^2$$

$$= 4\mu J + 16\mu J$$

$$= 20\mu J$$

Note: The given $i = 240e^{-10t}$ leaves us with $v = v_1(t=\infty) + v_2(t=\infty) = 0V$. We must have $v = 0V$ for the calculation of the trapped energy. Otherwise, we also have stored energy $= \frac{1}{2}C_{eq}v^2$.

A different i might not have resulted in $v=0$ as $t\to\infty$.