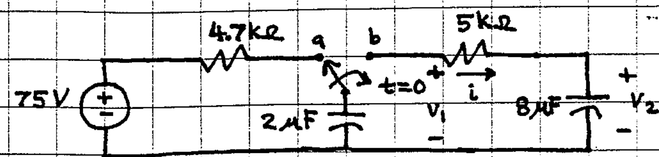
ex:



$v_2(t \le 0) = 0V$

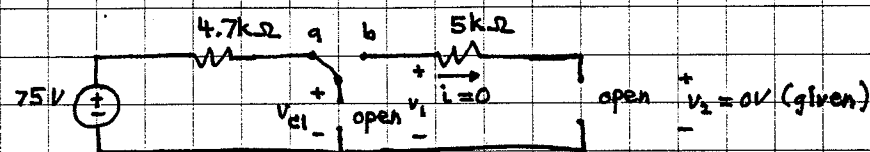
The switch in the circuit has been in position 'a' for a long time before it changes to position 'b' at $t=0$.

a) Find i, v_1, and v_2 for $t \ge 0^+$.

At $t=0^-$ we have $\frac{dv_c}{dt}=0$ both C's.

$\therefore i_c=0$ for both C's. $\therefore$ C's look like open circuits

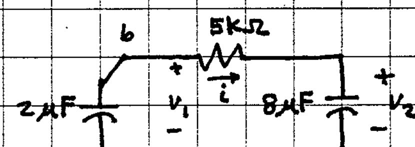
$t=0^-$



Since no current flows on the left side, there is no V-drop across the 4.7kΩ R. Thus, $v_{c1} = 75V$.

When we throw the switch at $t=0$, the left side 75V and 4.7kΩ no longer affect i, v_1, and v_2:

$t \ge 0^+$

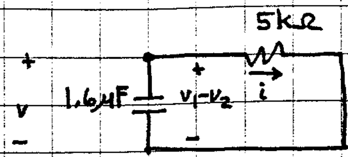


We combine the two C's in series into one $C_{eq} = 2\mu F \| 8\mu F$;

$$C_{eq} = \frac{2\mu F \cdot 8\mu F}{2\mu F + 8\mu F} = \frac{16}{10}\mu F = 1.6\,\mu F.$$

The initial voltage on C_{eq} is $v_1(0^+) - v_2(0^+) = v_1(0^-) - v_2(0^-)$, since V across C's cannot change instantly. (with $v_1(0^+) = 75V$, $v_2(0^+) = 0V$, $v_1(0^-) = 75V$, $v_2(0^-) = 0V$)

Now our equivalent circuit is an RC with initial voltage $v_{c1} - v_{c2} = 75V - 0 = 75V$:



We sum currents flowing out of node between R & C, (instead of summing V's around loop as we did for RL problems).

In other words, we equate the i in the R & C.

$$-i = C\frac{d\,v_1-v_2}{dt} = C\frac{dv}{dt} \quad \text{and} \quad i = \frac{v_2-v_1}{R} = \frac{v}{R}$$

$$\therefore \quad \frac{v}{R} = -C\frac{dv}{dt} \quad \text{or} \quad \frac{v}{R} + C\frac{dv}{dt} = 0\text{ A}$$

As shown on p.287 of text, the solution is

$$v(t) = v(t=0^+)\,e^{-t/RC_{eq}} = 75\text{V}\cdot e^{-t/5\text{k}\Omega\cdot 1.6\mu\text{F}}$$
$$= 75\text{V}\cdot e^{-t/8\text{ ms}}$$

$$i(t) = \frac{v(t)}{R} = \frac{75\text{V}\,e^{-t/8\text{ms}}}{5\text{k}\Omega} = 15\text{ mA}\,e^{-t/8\text{ ms}}$$

Now that we have $i(t)$, we can integrate $i(t)$ to find $v_1(t)$ and $v_2(t)$.

$$-i(t) = \overset{2\mu F}{C_1}\frac{dv_1}{dt} \qquad i(t) = \overset{8\mu F}{C_2}\frac{dv_2}{dt}$$

$$-\int_{t'=0^+}^{t'=t} i(t')\,dt' = C_1\int_{v_1(t=0^+)}^{v_1(t)} dv_1 \qquad \int_{t'=0^+}^{t'=t} i(t')\,dt' = C_2\int_{v_2(t=0^+)}^{v_2(t)} dv_2$$

$$\underset{60\text{V}}{\left(\frac{--8\text{ms}\cdot 15\text{mA}}{2\mu\text{F}}\right)} e^{-t'/8\text{ms}}\Big|_{t'=0}^{t'=t} = v_1(t) - v_1(0^+) \qquad \underset{-15}{\left(\frac{-8\text{ms}\cdot 15\text{mA}}{8\mu\text{F}}\right)} e^{-t'/8\text{ms}}\Big|_{t'=0}^{t'=t} = v_2(t) - v_2(0^+)$$

$$v_1(t) = 60\text{V}\left(e^{-t/8\text{ms}} - 1\right) + 75\text{V} \qquad v_2(t) = -\quad \text{V}\left(e^{-t/8\text{ms}} - 1\right)$$