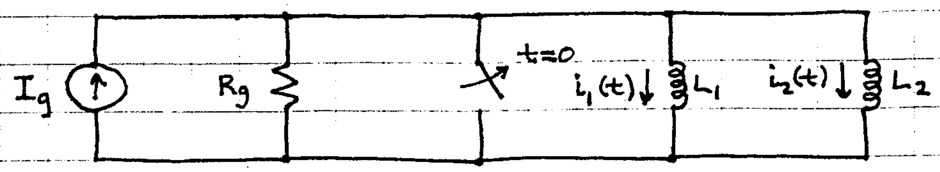
ex:



No energy stored in L_1 and L_2 when switch opens

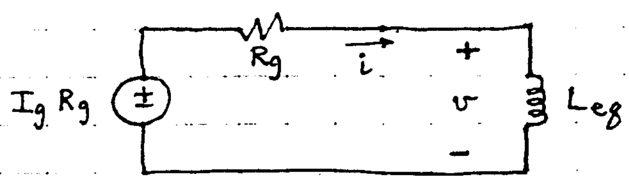
a) Find $i_1(t\geq 0)$ and $i_2(t\geq 0)$. b) Find $i_1(t\to\infty)$ and $i_2(t\to\infty)$

ans: a) $i_1(t\geq 0) = I_g \dfrac{L_2}{L_1+L_2}\left(1-e^{-t/\tau}\right)$ where $\tau \equiv L_1 \| L_2$

$i_2(t\geq 0) = I_g \dfrac{L_1}{L_1+L_2}\left(1-e^{-t/\tau}\right)$ " "

b) $i_1(t\to\infty) = I_g \dfrac{L_2}{L_1+L_2}$ $i_2(t\to\infty) = I_g \dfrac{L_1}{L_1+L_2}$

sol'n: a) Take Thevenin equiv of I_g and R_g on left.
Solve for v across L's by replacing L's with equivalent L:



circuit for $t\geq 0$

L's in parallel give $L_{eg} = L_1 \| L_2 = \dfrac{L_1 L_2}{L_1+L_2}$.

(L's in parallel are like R's in parallel in terms of the formula we use.)

Now we use the general solution for $v(t\geq 0)$:

$$v(t\geq 0) = v(t\to\infty) + [v(0^+) - v(t\to\infty)]\,e^{-t/(L_{eg}/R_g)}$$

time constant

To find $v(0^+)$, we use $i_1(0^+) = i_1(0^-)$ and $i_2(0^+) = i_2(0^-)$. But $i_1(0^-) = i_2(0^-) = 0$ since no energy is stored in L_1 and L_2 at $t=0$.

a) cont.

Since $i_1(0^+)$ and $i_2(0^+) = 0$, we must have no current through R_g at $t = 0^+$.

$\therefore$ At $t=0^+$, we have no v drop across R_g.

$$\therefore \quad v(t=0^+) = V_{Th} = I_g R_g$$

For $v(t\to\infty)$ we observe that the L's act like wires, and $v(t\to\infty) = 0$.

Plugging into the general soln gives

$$v(t\geq 0) = I_g R_g \, e^{-t/(L_{eq}/R_g)}$$

Note: The time constant for circuit with L and R is L_{eq}/R_{Thev}. Taking the Thevenin equivalent always gives the needed R.

Now we can also write down a formula for $i(t) = i_1(t) + i_2(t)$ for $t \geq 0$:

$$i(t\geq 0) = i(t\to\infty) + [i(0^+) - i(t\to\infty)]\, e^{-t/(L_{eq}/R_g)}$$

Note: All i's and v's have same time constant.

We know $i(0^+) = i_1(0^+) + i_2(0^+) = i_1(0^-) + i_2(0^-) = 0$

At $t\to\infty$, the L's act like wires, giving $i = I_g$.

$$\therefore \quad i(t\geq 0) = I_g\left[1 - e^{-t/(L_{eq}/R_g)}\right]$$

Now we determine how $i(t\geq 0)$ is divided between the two L's to give $i_1(t\geq 0)$ and $i_2(t\geq 0)$.

a) cont.

Since both L's have same V across them, we have

$$v = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} \qquad \therefore \frac{\frac{di_1}{dt}}{\frac{di_2}{dt}} = \frac{L_2}{L_1}$$

Now we calculate currents:

$$i_1(t) = \int \frac{di_1}{dt}\, dt = \int \frac{L_2}{L_1} \frac{di_2}{dt}\, dt = \frac{L_2}{L_1} \int di_2$$

or $i_1(t) = \frac{L_2}{L_1} i_2(t)$

Also, $i_1(t) + i_2(t) = i(t)$.

Solving these two eq'ns gives

$$i_1(t) = \frac{L_2}{L_1+L_2} i(t)$$

$$i_2(t) = \frac{L_1}{L_1+L_2} i(t)$$

Thus,

$$i_1(t \geq 0) = I_g \frac{L_2}{L_1+L_2}\left(1 - e^{-t/(L_{eq}/R_g)}\right)$$

$$i_2(t \geq 0) = I_g \frac{L_1}{L_1+L_2}\left(1 - e^{-t/(L_{eq}/R_g)}\right)$$

b) At $t \to \infty$ we have $e^{-t/(L_{eq}/R_g)} \to 0$.

$$\therefore i_1(t \to \infty) = I_g \frac{L_2}{L_1+L_2}$$

$$i_2(t \to \infty) = I_g \frac{L_1}{L_1+L_2}$$