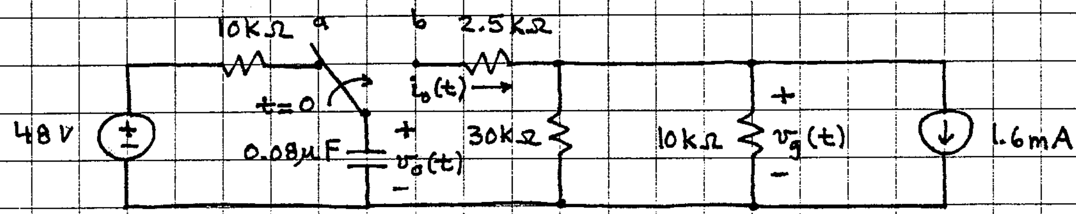
ex:




Find a) $v_o(t \geq 0)$ b) $i_o(t \geq 0)$ c) $v_g(t \geq 0)$ d) $v_g(0^+)$

ans: a) $v_o(t \geq 0) = -12 + 60e^{-t/0.8ms}$ V

b) $i_o(t \geq 0) = 6e^{-t/0.8ms}$ mA

c) $v_g(t \geq 0) = -12 + 45e^{-t/0.8ms}$ V

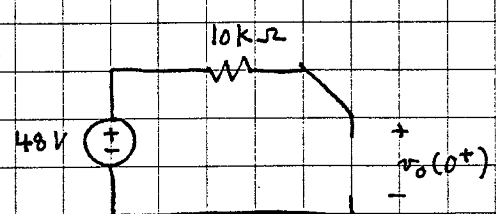
d) $v_g(0^+) = 33$V

soln: a) Step 1: Find v's for capacitors for $t=0^-$
" i's " inductors " "

(Here, we just have one C.)

tool: $i$ (for C) = 0 at $t=0^-$ open circuit
$v$ ( " L) = 0 " " wire

$t=0^-$ circuit: (can ignore right side of circuit)



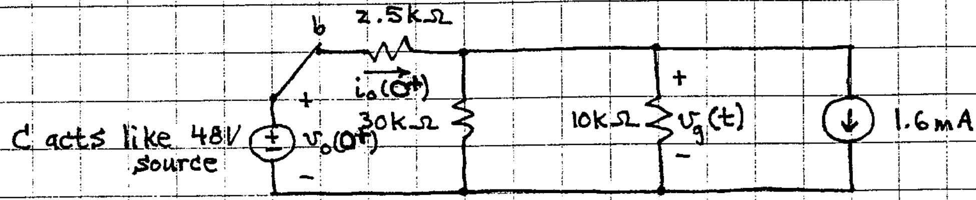

Since no current flows, V drop across 10kΩ = 0V.

$\therefore v_o(0^+) = 48V$

Step 2: V on C cannot change instantly
$i$ " L " " "

Use ↑ to find v's and i's at $t=0^+$.

$t = 0^+$ circuit: (can ignore left side of circuit)




Note: At $t=0^+$ we model C as V source
" " " " L " i source

Clearly, $v_o(0^+) = 48V$.

We use node voltage method to find $v_g(t)$:

$$\frac{v_g - 48V}{2.5k\Omega} + \frac{v_g}{30k\Omega} + \frac{v_g}{10k\Omega} + 1.6\,mA = 0\,A$$

$$\left(\text{or } \frac{v_g}{2.5k\Omega \| 30k\Omega \| 10k\Omega} = \frac{48V}{2.5k\Omega} - 1.6\,mA\right)$$

mult by 30kΩ both sides

or $(v_g - 48V)\,12 + v_g + 3v_g + 1.6mA\,(30k\Omega) = 0$

or $16\,v_g = 48V \cdot 12 - 1.6(30)\,V = 48V \cdot 12 - 16(3)\,V$

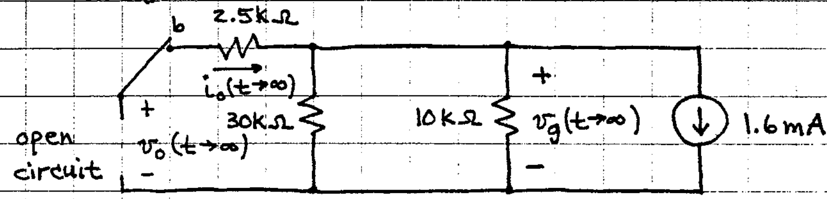
or $v_g = 3 \cdot 12 - 3 = 33V$

Thus, $v_g(t=0^+) = 33V$ (answer to "d")

So $i_o(t=0^+) = \frac{48V - 33V}{2.5k\Omega} = \frac{15V}{2.5k\Omega} = 6\,mA$

Step 3: Find v's for capacitors for $t \to \infty$
" i's " inductors " "
Use these to find other i's and v's in circuit.
use tool: $i(\text{for } C) = 0$ and $v(\text{for } L) = 0$ at $t \to \infty$

$t \to \infty$ circuit:




$i_o(t\to\infty) = 0$ since C is open circuit

Since $i_o(t\to\infty) = 0$, we have no V-drop across 2.5kΩ resistor.

$$\therefore v_o(t\to\infty) = v_g(t\to\infty)$$

To find $v_g(t\to\infty)$ we observe that we have current divider consisting of the 30kΩ and 10kΩ resistors. The voltage, $v_g(t\to\infty)$, will be given by $-1.6mA \cdot 10k\Omega \| 30k\Omega$.

$$v_g(t\to\infty) = -1.6\,mA \cdot 10k\Omega \cdot 1\|3 = -16V \cdot \frac{3}{4} = -12V$$

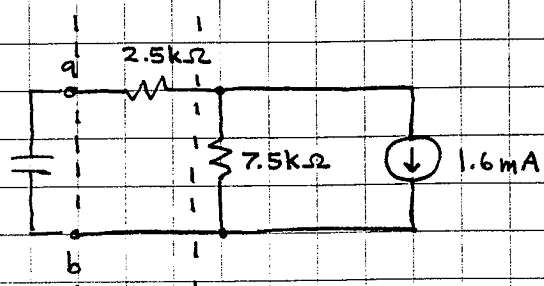
Step 4: Take Thevenin equivalent of circuit without C or without L. $R_{Th}$ is R value for time constant.

Combine series or parallel C's or L's to obtain equivalent $C_{eq}$ or $L_{eq}$ for time constant.

For RC circuit, time constant $= R_{Th}\, C_{eq}$

For RL circuit, " " $= L_{eq} / R_{Th}$

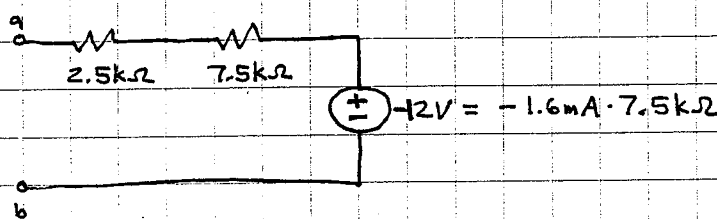
Here, we have $10k\Omega \| 30k\Omega = 10k\Omega \cdot 1\|3 = 7.5\,k\Omega$ as resistance across 1.6 mA.



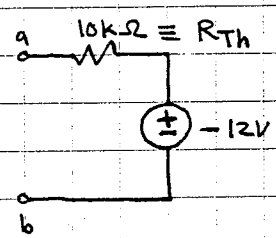

Thev equiv ⇒

Take Thev equiv ⇒
and add 2.5kΩ

or




or




Time constant $= R_{Th} \cdot C = 10\text{k}\Omega \cdot 0.08\mu\text{F} = 0.8 \text{ ms}$

Step 5: Use general form of sol'n for all $i$'s and $v$'s in the circuit:

$$i(t \geq 0) = i(t \to \infty) + \left[i(0^+) - i(t \to \infty)\right] e^{-t/\tau}$$

$$v(t \geq 0) = v(t \to \infty) + \left[v(0^+) - v(t \to \infty)\right] e^{-t/\tau}$$

where $\tau \equiv$ time constant $= RC$ or $L/R$

Note: all $i$'s and $v$'s in the circuit have <u>same time constant</u>.

Now we can write down the formulas given in the answer to the problem.