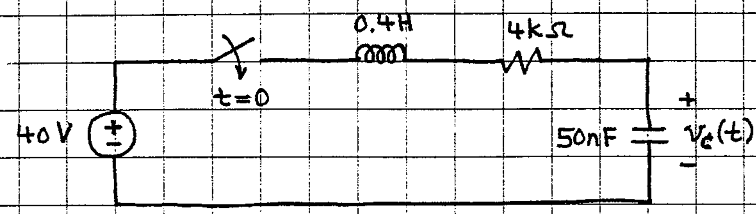

Initial stored energy = 0. Find $v_c(t \geq 0)$.

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ans: $v_c(t \geq 0) = 40 - 40e^{-5kt}(\cos 5kt + \sin 5kt)$

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sol'n: For $t > 0$ we must have a general series RLC differential eq'n with a driving term of 40V:

$$\frac{d^2 v_c}{dt} + \frac{R}{L}\frac{dv_c}{dt} + \frac{v_c}{LC} = \frac{40V}{LC} \qquad \text{(see text eq'n 8.66 p. 379)}$$

To determine what general form of sol'n to use, we first find the roots, s_1 and s_2, of the characteristic eq'n:

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \qquad \alpha = -\frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

We have $\alpha = +\frac{4k}{2(0.4)}$ rad/s $= +5k/s$ (I usually leave out "rad")

$$\omega_0^2 = \frac{1}{\sqrt{(0.4)(50n)}^2} = \frac{1}{\sqrt{20n}^2} = 50M/s^2$$

$$\sqrt{\alpha^2 - \omega_0^2} = \sqrt{25M - 50M}\ /s = j5k/s$$

$$\therefore\ s_1, s_2 = -5k \pm j5k\ /s \qquad \omega_d = \sqrt{\omega_0^2 - \alpha^2} = 5k/s$$

Complex roots ⇒ underdamped sol'n:

$$v_c = V_f + B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$$

where $V_f \equiv v_c(t \to \infty) = 40V$ (Reason: as $t \to \infty$ L looks like wire; C charges to 40V since eventually no more current flows and $v_{4k\Omega} = 0$.)

Now we have to match $v(t=0^+)$ and $\left.\frac{dv(t)}{dt}\right|_{t=0^+}$ to initial circuit $v_C(t=0^+)$ and $i_L(t=0^+)$.

Note: Text uses $t=0$ instead of $t=0^+$. Strictly speaking we should say $t=0^+$ in case i_R changes when we throw the switch. This will become an issue when we use state-variable techniques. We should also be careful to say our sol'n is for $t>0$ rather than $t \geq 0$. In this problem, i_R stays the same when we close the switch, and our $v(t)$ happens to be valid for $t=0$, too.

Since initial stored energy $= 0$, we have $v_C(t=0^-) = 0V$ and $i_L(t=0^-) = 0A$.

But $v_C(t=0^+) = v_C(t=0^-)$ since v_C can't change instantly.
$i_L(t=0^+) = i_L(t=0^-)$ " i_L " " "

$\therefore$ $v_C(t=0^+) = 0V$ $\quad$ $i_L(t=0^+) = 0A$

First initial condition:

we use 0, not 0^+, for this calculation; i.e. $0^+=0$ here

$$v_C(t=0^+) = 0V = 40V + B_1 e^{-\alpha \cdot 0} \cos \omega_d \cdot 0 + B_2 e^{-\alpha \cdot 0} \sin \omega_d \cdot 0$$
$$= 40V + B_1 \cdot 1 \cdot 1 + B_2 \cdot 1 \cdot 0$$
$$= 40V + B_1$$

$\therefore B_1 = -40V$

Second initial condition: $\left. C\frac{dv_C}{dt}\right|_{t=0^+} = i_C(t=0^+) = i_L(t=0^+) = 0A$

C, L in series, same i

$$\left.\frac{dv_C}{dt}\right|_{t=0^+} = 0 \text{ V/s} = \left. \begin{aligned} &B_1[(-\alpha)\cos \omega_d t + \omega_d(-\sin \omega_d t)]e^{-\alpha t} \\ &+ B_2[(-\alpha)\sin \omega_d t + \omega_d(\cos \omega_d t)]e^{-\alpha t} \end{aligned} \right|_{t=0^+}$$

or $\quad 0\ V/s = B_1\left[(-\alpha)\cos\omega_d\cdot 0 + \omega_d(-\sin\omega_d\cdot 0)\right]e^{-\alpha\cdot 0}$

$\quad + B_2\left[(-\alpha)\sin\omega_d\cdot 0 + \omega_d(\cos\omega_d\cdot 0)\right]e^{-\alpha\cdot 0}$

$\quad = B_1\left[(-\alpha)\cdot 1 + \omega_d\cdot 0\right]\cdot 1$

$\quad + B_2\left[(-\alpha)\cdot 0 + \omega_d\cdot 1\right]\cdot 1$

$0\ V/s = -\alpha B_1 + \omega_d B_2$

$\quad" = -5k/s\cdot(-40V) + 5k/s\ B_2$

$\therefore \quad B_2 = -40V$

$$\boxed{\therefore\ v(t\geq 0) = 40V - 40V e^{-5kt}\left(\cos 5kt + \sin 5kt\right)}$$