TOOL: Series and parallel *RLC* circuits may be solved by a step-by-step procedure outlined below in (a)-(d). Figs. 1 and 2 illustrate series and parallel *RLC* circuits. Note that the circuitry to which the *L* and *C* are connected is converted to a Thevenin equivalent, which turns all the *R*'s in the circuit into a single *R*.

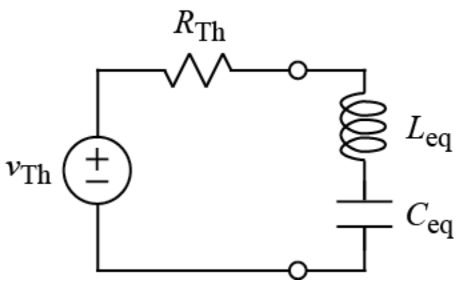


Fig. 1. Series *RLC* circuit.

$$\alpha = \frac{R_{Th}}{2L_{Eq}}$$

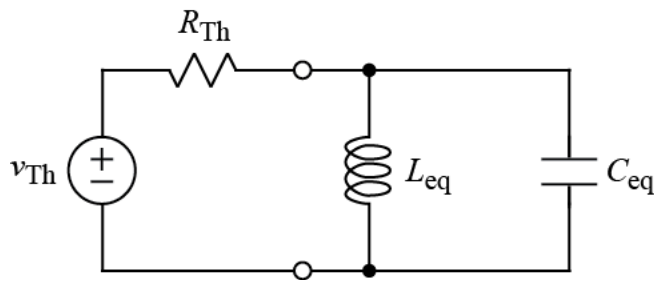


Fig. 2. Parallel *RLC* circuit.

$$\alpha = \frac{1}{2R_{Th}C_{Eq}}$$

Series or parallel *RLC*:

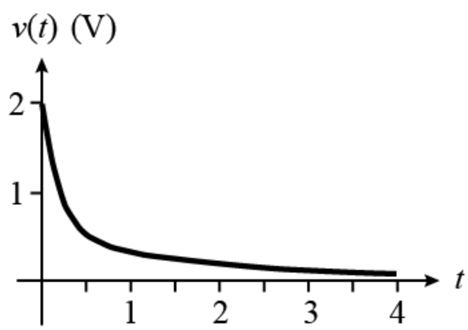
$$\omega_0^2 = \frac{1}{LC}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Over-damped: $s_{1,2}$ real, negative, distinct

$v(t > 0) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3$ also applies to $i(t > 0)$

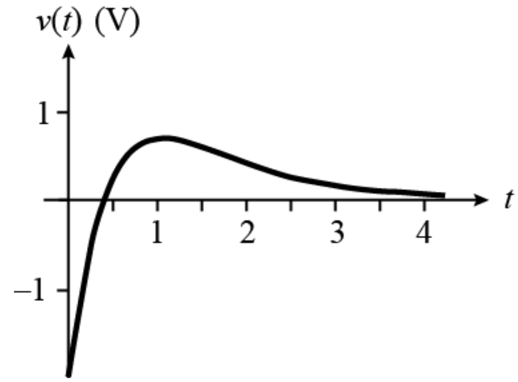
$$v(t = 0^+) = A_1 + A_2 + A_3 \qquad \left.\frac{dv(t)}{dt}\right|_{t=0^+} = A_1 s_1 + A_2 s_2$$



Critically-damped: s1 = s2 = –α real, negative, repeated

$v(t>0) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t} + A_3$ also applies to $i(t>0)$

$v(t=0^+) = A_1 + A_3$ $\quad \left.\frac{dv(t)}{dt}\right|_{t=0^+} = A_1(-\alpha) + A_2$

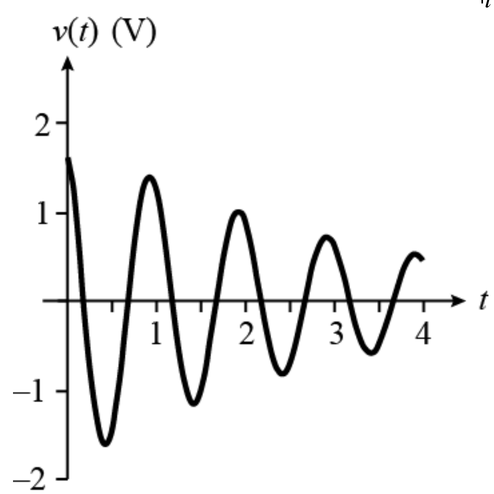


Under-damped: $s_{1,2} = -\alpha \pm j\omega_d$ complex, distinct

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ damping frequency

$v(t>0) = A_1 e^{-\alpha t}\cos(\omega_d t) + A_2 e^{-\alpha t}\sin(\omega_d t) + A_3$ also applies to $i(t>0)$

$v(t=0^+) = A_1 + A_3$ $\quad \left.\frac{dv(t)}{dt}\right|_{t=0^+} = A_1(-\alpha) + A_2\omega_d$



Final value:

$t \to \infty$: Use L = wire, C = open.
$A_3 = v(t \to \infty)$ also applies to $i(t > 0)$

Initial value:

$t = 0^-$: Find $i_L(t = 0^-)$ and $v_C(t = 0^-)$.

$t = 0^+$: Find $v(t = 0^+)$ (or whatever variable is being solved for)

Use $i_L(t = 0^+) = i_L(t = 0^-)$ and $v_C(t = 0^+) = v_C(t = 0^-)$.

Use L = i-source of value $i_L(t = 0^+)$, C = v-source of value $v_C(t = 0^+)$.

Find $v(t = 0^+)$ from circuit.

Equate $v(t = 0^+)$ from circuit with $v(t = 0^+)$ for symbolic solution.

Initial value of derivative:

$t > 0$: Model L as i-source of value i_L and C as v-source of value v_C.

Write expression for $v(t)$ (or whatever variable is being solved for) in terms of i_L and v_C.

$t = 0^+$: Take derivative of both sides of this expression for $v(t)$ (or whatever variable is being solved for) and evaluate at $t = 0^+$.

Use $\left.\dfrac{dv_C(t)}{dt}\right|_{t=0^+} = \dfrac{i_C(t=0^+)}{C}$ and/or $\left.\dfrac{di_L(t)}{dt}\right|_{t=0^+} = \dfrac{v_L(t=0^+)}{L}$

Complete the calculation of $\left.\dfrac{dv(t)}{dt}\right|_{t=0^+}$ from circuit.

Equate $\left.\dfrac{dv(t)}{dt}\right|_{t=0^+}$ from circuit with $\left.\dfrac{dv(t)}{dt}\right|_{t=0^+}$ for symbolic solution.