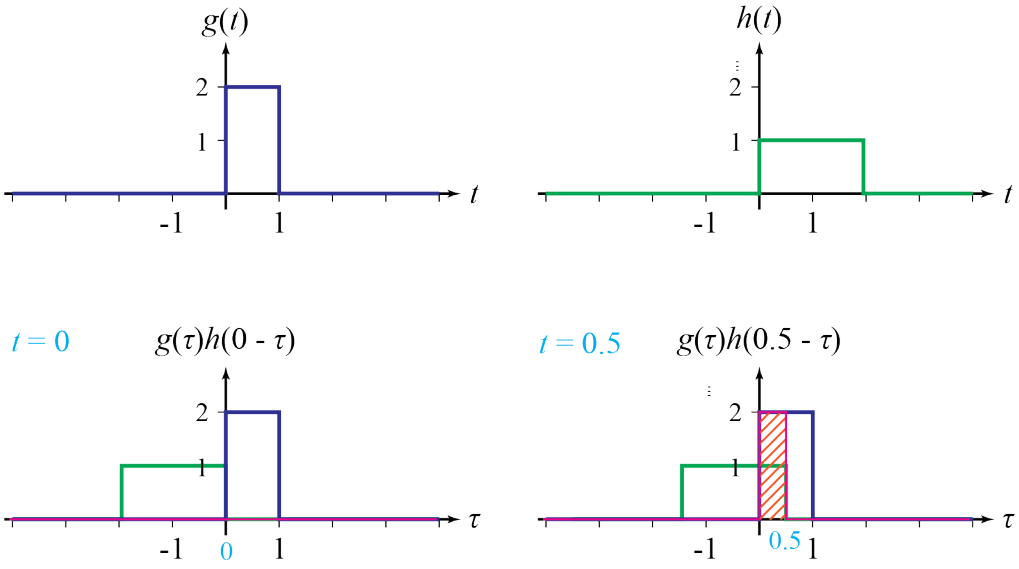
By: Neil E. Cotter

TOOL: The convolution of two functions is the integral of the first function multiplied by a horizontally flipped and time shifted copy of the second function.

$$f(t) = g(t) * h(t) \equiv \int_{-\infty}^{t} g(\tau)h(t-\tau)d\tau$$

The calculation of the convolution integral must be carried out for each value of t. For a given value of t, the origin of $h(t)$ is located at τ, $h(t)$ is reversed in time, and $h(t)$ becomes a function of τ. Next, the product of $g(t)$ and $h(t - \tau)$ is computed. The area under the product curve is the value of the convolution for t. The figures below illustrate the process of computing the value of the convolution at various values of t. for rectangular-shaped functions, $g(t)$ and $h(t)$.

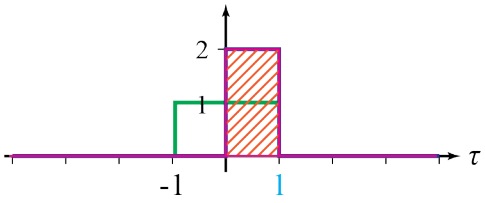
NOTE: **See the other CTool pages on convolution for a much easier method** of computing convolutions based on differentiation and impulses.



Area under product = 0

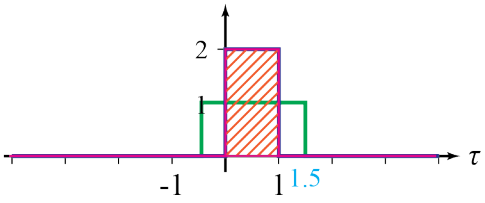
Area under product = 1

$t = 1$ $g(\tau)h(1 - \tau)$



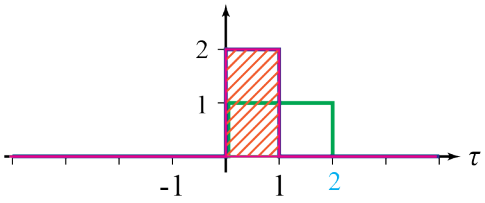
Area under product = 2

$t = 1.5$ $g(\tau)h(1.5 - \tau)$



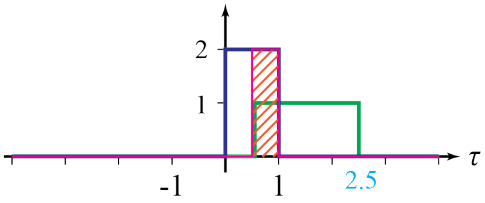
Area under product = 2

$t = 2$ $g(\tau)h(2 - \tau)$



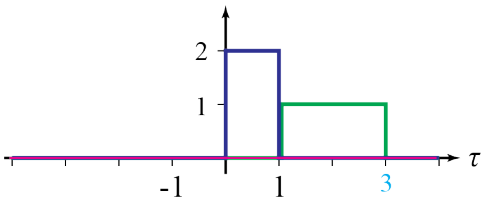
Area under product = 2

$t = 2.5$ $g(\tau)h(2.5 - \tau)$



Area under product = 1

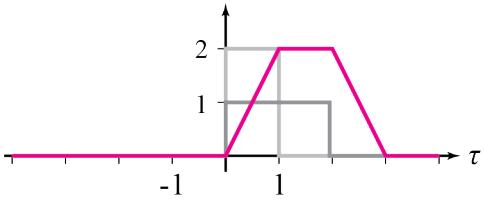
$t = 3$ $g(\tau)h(3 - \tau)$



Area under product = 0

Final result:

$f(t) = g(t) * h(t)$



REF: Ronald Bracewell, *The Fourier Transform and its Applications*, 2nd Ed., New York, NY: McGraw-Hill, 1978.