EX: Fig. 1 below shows an example of using differentiation and integration to find the convolution of a triangular function and a rectangle function. Notes, line by line on the solution:

① The convolved functions are shown with the answer left blank.

② Either of the functions in Line ① 1 may be differentiated, but differentiating the rectangle function is simpler. The rectangle is a step up and a step down, which translates to a positive impulse function of area +1 and a negative impulse function of area −1.

③ The impulses are processed separately. Each impulse creates a copy of the trianglular function, shifted by the position of the impulse and multiplied by the area of the impulse.

④ The second impulse replicates the triangular function, upside down and shifted to the right by one unit. The triangles from Lines ③ and ④ will be summed.

⑤ The triangles from Lines ③ and ④ are summed.

⑥ To integrate from $-\infty$ to t, we are calculating the area to the left of a conceptual vertical line at time t. The idea of graphical math is to use simple sketching rules and areas of simple geometric shapes to get the desired final answer in the lower right of the diagram. For the integrals of the triangles, the area of the first triangle is one square unit. Thus, when t reaches the end of the first triangle, the value $x(t) * h(t)$ will be 1.

The integral of a triangle or any sloped line will be a quadratic shape. We sketch the quadratic segment using the total area of the triangle and the values at the start and end of the triangle.

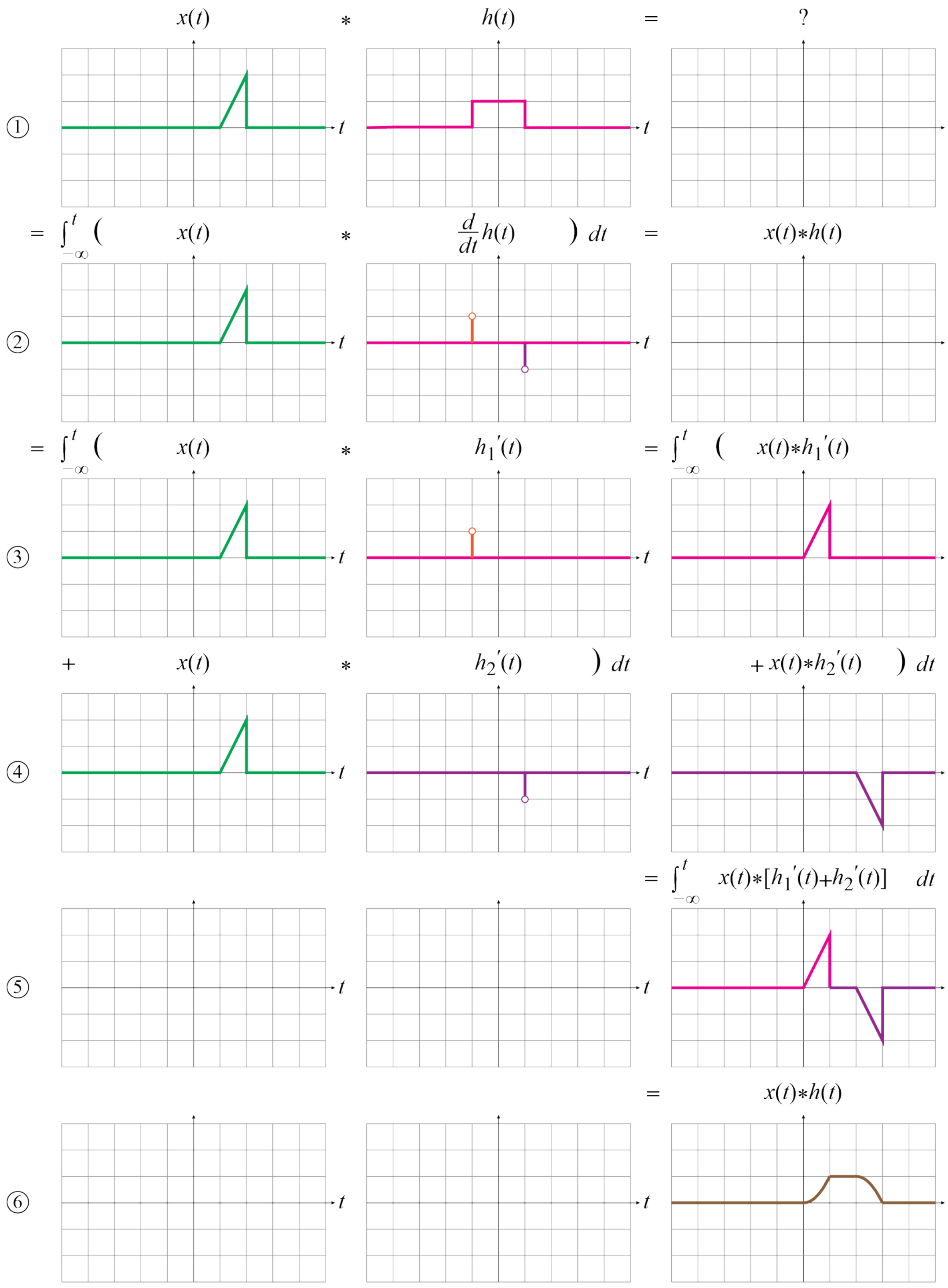

Fig. 1. Convolution by differentiation, convolution, integration.

At the start of the first triangle where $t = 0$, the value of $x(t) * h'(t)$ is 0, and at the end of the first triangle where $t = 1$, the value of $x(t) * h'(t)$ is 2. These values are the slope of the quadratic segment at the start and end. Thus, the first quadratic segment of the integral starts out horizontal (slope = 0) and ends with a slope of 2. So long as we account for the nature of the slopes at the ends of the quadratic segment, our sketch is adequate. The exact value of the curve between endpoints may be approximate.

By symmetry, the integral of the second triangle gives the same shape as the integral of the first segment but upside down, and it must start from the value of the integral at $t = 2$.

With two functions that have finite areas, the convolution must have finite area, meaning the convolution value must return to zero as $t \to \infty$.