
EX: Fig. 1 below shows an example of replication by impulses to find the output of a digital filter as the convolution of sampled input, $x[n]$, and filter input response, $h[n]$. The Rosetta Stone method provides a straightforward way of visualizing the output of a digital filter as a series of shifts and adds of scaled impulse inputs, $x[n]$. By commutativity, the output of the filter may also be found as a series of shifts and adds of scaled impulse response, $h[n]$. Notes, line by line on the solution:

① Statement of problem with the answer left blank.

② Each of Lines 2 thru 4 has one impulse from $h[n]$. Because it leads to simpler shifts and multiplies, we decompose $h[n]$ rather than $x[n]$ here. Note that this decomposition relies on the principle of superposition. In Line 2, the first impulse is unit area and located at the origin, so it creates an unaltered copy of $x[n]$. The impulse response is causal since all impulses occur at $t = 0$ or later.

③ Second impulse in $h[n]$, with area equal to one, occurs at $n = 1$, creating a copy of $x[n]$ shifted to the right by one time unit.

④ Third impulse in $h[n]$, with area equal to -1, occurs at $n = 2$, creating a copy of $x[n]$ shifted to the right by one time unit and turned upside-down.

⑤ The three sets of impulses on the right side of Lines 2 thru 4 are summed. The impulses at $t = 0$ cancel out. Impulses occur at each sample time but may be omitted if they have zero area.

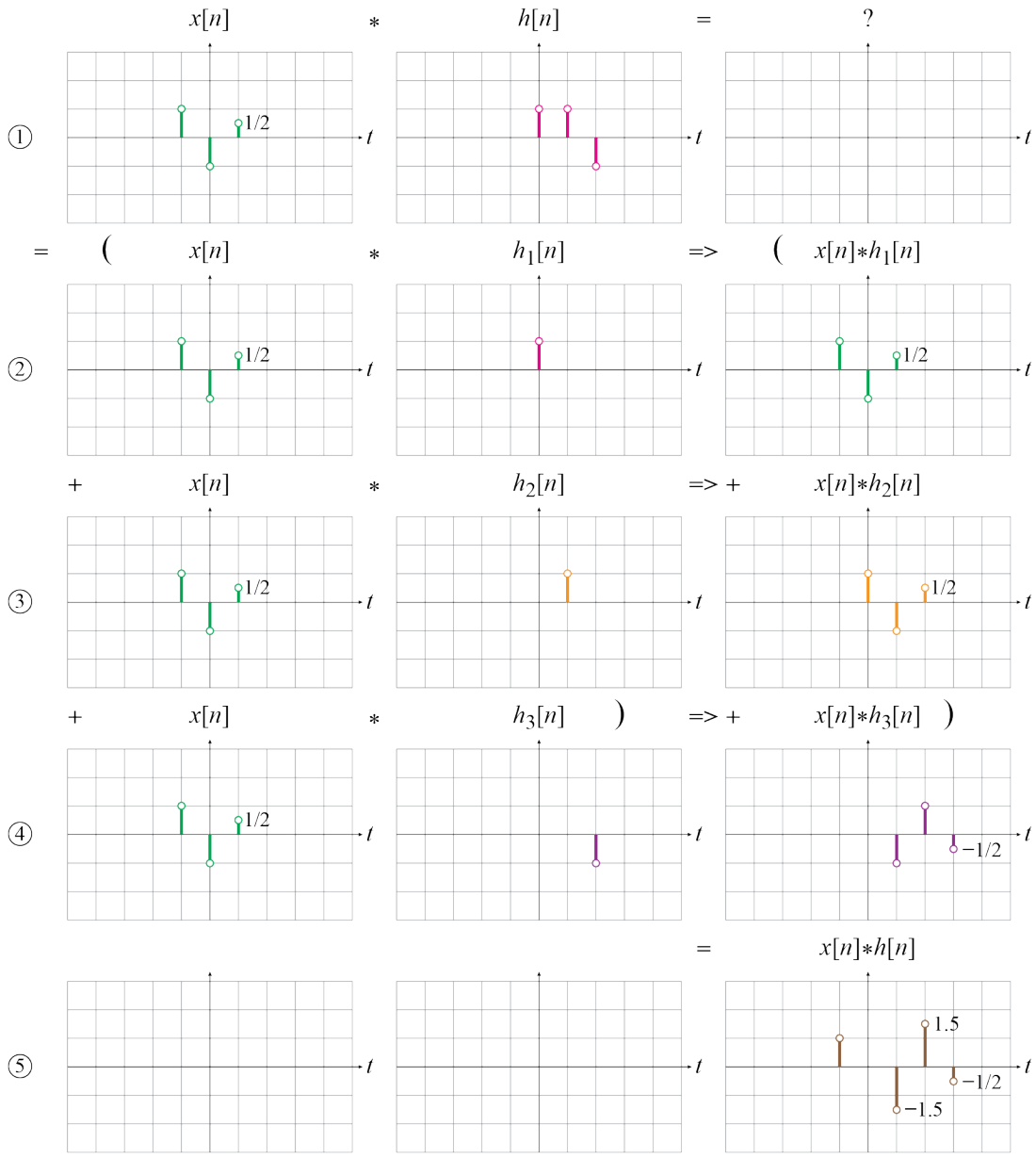

Fig. 1. Convolution by impulse replication and superposition.