EX: Fig. 1 below shows an example of using differentiation and integration to find the convolution of two functions. Notes, line by line on the solution:

① The convolved functions are shown with the answer left blank.

② Either of the functions in Line ① 1 may be differentiated, but differentiating the function on the right is a bit simpler. The derivative of a step is an impulse or area equal to the height of the step. Thus, we have three impulses for the derivative, shown in Lines 2 thru 5.

The impulses are processed separately. Each impulse creates a copy of the function on the left. The copy is shifted by the position of the impulse and multiplied by the area of the impulse. The first impulse is unit area and located at the origin. The first convolution just replicates the waveform on the left.

③ The second impulse replicates the function on the left shifted to the right by one unit, inverted, and scaled by two. The shift and scaling correspond to the impulse on the right being located at $t = 1$ and having an area of minus two.

④ The third impulse replicates the function on the left shifted to the right by two units. The shift and scaling correspond to the impulse on the right being located at $t = 2$ and having an area of one.

⑤ The right sides of Lines 2 thru 4 sum to give the graph on the right of Line 5.

⑥ The final answer is the integral of the result in Line 5. To integrate from $-\infty$ to t, we are calculating the area to the left of an imagined vertical line at time t. The idea of graphical math is to use simple sketching rules and areas of simple geometric shapes to get the desired final answer in the lower right of the diagram. For the integrals of the steps, we get a sloped line where the slope is the height of the step. The integrals of impulses are steps of heights equal to the areas of the impulses.

With two functions that have finite areas, the convolution must have finite area, meaning the convolution value must return to zero as $t \to \infty$.

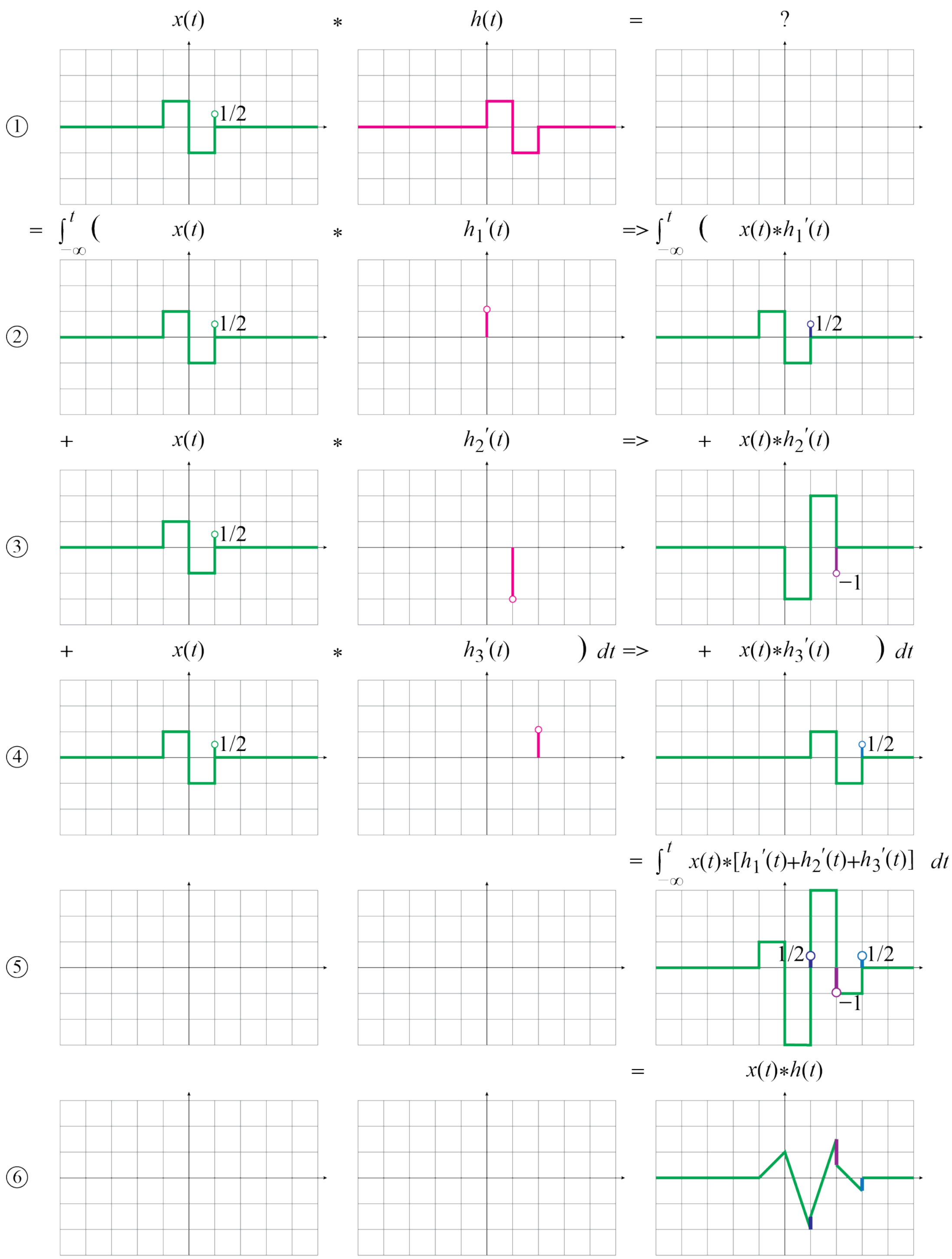

Fig. 1. Convolution by differentiation, convolution, integration.