EX: Plot the convolution function, $h(t) = f(t) * g(t)$, where

$$f(t) = \delta(t+1) - \tfrac{1}{2}[u(t-1) - u(t-2)]$$

$$g(t) = -u(t) + u(t-1) + \tfrac{1}{2}\delta(t-2)$$

Use the convolution worksheet found in CTools: Rosetta Stone Method: Convolution: worksheet to solve the problem graphically.

SOLN: See Fig. 1 on following page. Line by line explanation of Fig. 1:

① The convolved functions are shown with the answer left blank.

② The function on the left, $f(t)$, is decomposed into an impulse at $t = -1$, $f_1(t)$ on Line 2, and a rectangle function, $f_2(t)$, from $t = 1$ to 2 for Lines 3 and 4. The individual convolutions of either of the functions in Line ① 1 may be differentiated, but $f(t)$ on the left side was arbitrarily chosen for decomposition.

③ Function $f_2(t)$ is processed further in Lines 3 and 4 by taking its derivative and putting the two resulting impulses, $f'_{21}(t)$ and $f'_{22}(t)$ on separate lines.

④ The impulses on Lines 2 thru 4 are convolved with $g(t)$ individually, giving the results in the right column. Since all the convolutions are with impulses, the results in the right column are copies of $g(t)$ that are shifted and scaled according to the impulses in the first column.

⑤ Take note of the parentheses, plus signs, and integrals on Lines 2 thru 4. In equation form, we have the expression in Line 5:

$$h(t) = f_1(t) * g(t) + \int_{-\infty}^{t} [f'_{21} + f'_{22}] * g(t)dt$$

Line 5 shows the value of $h(t)$ by bringing down the waveform for $f_1(t) * g(t)$ from Line 2 and adding the integrals of Line 3 and Line 4. The integration is performed in place here, since the waveforms are essentially non-overlapping. A second page of the worksheet may used to do separate integral steps, if desired.

⑥ The final waveform on Lines 5 and 6 satisfies several consistency checks. First, the shape of $g(t)$ appears in the answer, which is expected since the impulse for $f_1(t)$ is positive and will make a copy of $g(t)$ under convolution. Second, the impulse in $g(t)$ will make a copy of the impulse and negative rectangle in $f(t)$. These shapes appear in the final answer, as expected. Third, the two rectangles in $f(t)$ and $g(t)$ have rectangles (constant segments of order 0) that, when convolved, will create a shape of order $0+0+1=1$, which is a sloped line. We actually get two sloped lines that form a triangle. We always get triangles when convolving rectangles.

See Fig. 1 on next page.

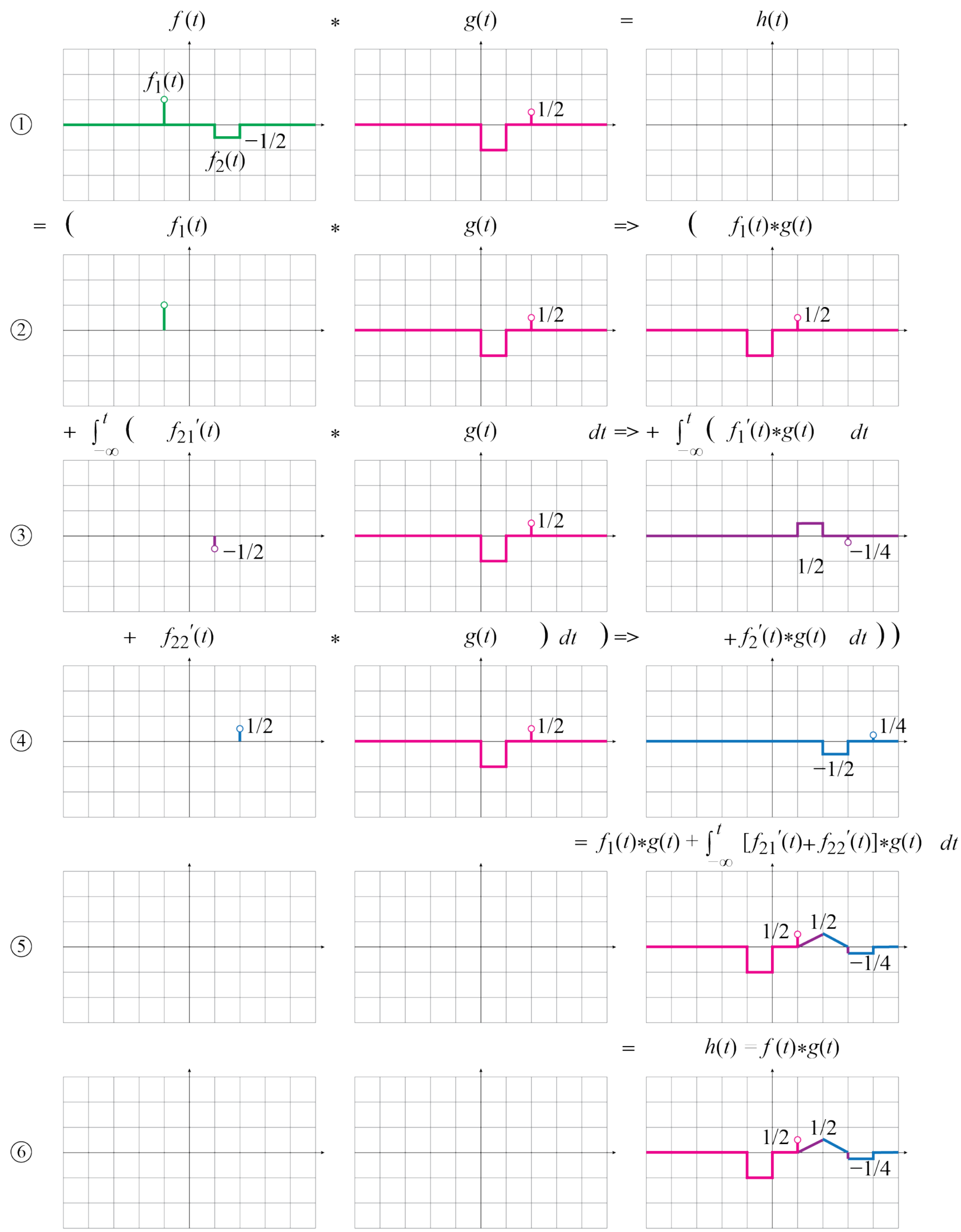

Fig. 1. Convolution by differentiation, convolution, integration.