

NOT'N: $f \equiv f(t), g \equiv g(t), h \equiv h(t),$
 $F \equiv F(f) \equiv \mathcal{F}\{f(t)\}, G \equiv G(f) \equiv \mathcal{F}\{g(t)\}, H \equiv H(f) \equiv \mathcal{F}\{h(t)\}$

TOOL: Convolution, as a binary mathematical operation, $*$, is:

associative:

$$(f * g) * h = f * (g * h)$$

commutative:

$$f * g = g * f$$

distributive:

$$f * (g + h) = f * g + f * h \quad \text{Note: } * \text{ is performed before } +$$

unital with identity $\delta(t)$ (i.e., convolution has identity function, $\delta(t)$):

$$f(t) * \delta(t) = f(t)$$

TOOL: Convolution of functions is isomorphic to multiplication of Fourier transforms and vice versa:

$$\mathcal{F}\{g * h\} = G \cdot H$$

$$\mathcal{F}\{f \cdot g\} = F * G$$

$$\mathcal{F}\{f * g * h\} = F \cdot G \cdot H$$

$$\mathcal{F}\{f * (g \cdot h)\} = F \cdot (G * H)$$

TOOL: Convolution of functions has the following behavior with respect to changes in sign and complex conjugates (i.e., asterisk superscript):

$$\mathcal{F}\{g(t)\} = G(f)$$

$$\mathcal{F}\{g(-t)\} = G(-f)$$

$$\mathcal{F}\{g^*(t)\} = G^*(-f)$$

$$\mathcal{F}\{g^*(-t)\} = G^*(f)$$

REF: Ronald Bracewell, *The Fourier Transform and its Applications*, 2nd Ed.,
 New York, NY: McGraw-Hill, 1978.