TOOL: The convolution of an exponential decay and a rectangle function may be sketched as an exponential growth and decay. Fig. 1 shows the functions being convolved.

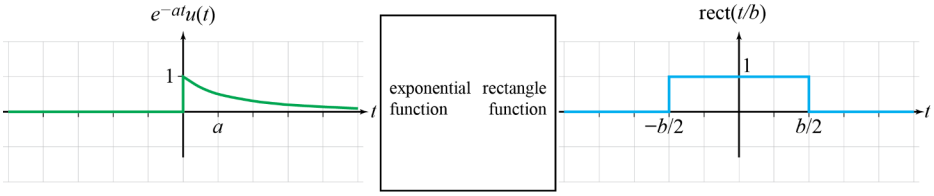
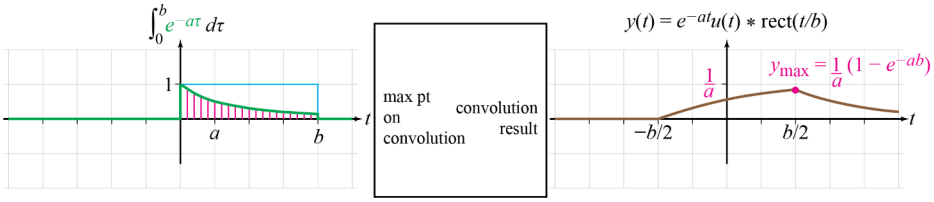


Fig. 1. Functions being convolved.

Fig. 2 shows the result of the convolution.



(a) (b)

Fig. 2. (a) Calculation of maximum value of convolution result.
(b) Convolution result.

Rather than computing the convolution integral each time, we may employ the following techniques to accurately sketch the convolution result, $y(t)$, as shown in Fig. 2:

1) Scaling the exponential or the rectangle function by a constant scales $y(t)$ by the same constant.

2) The curve for $y(t)$ starts to rise at $-b/2$ and ends at $b/2$.

3) The peak value of $y(t)$ is $y_{\max} = \dfrac{1}{a}(1 - e^{-ab})$

4) The shape of $y(t)$ is of the form e^{-at}. That is, it is an exponential with time constant $1/a$ for both the rise and decay.

$$y(t) = \begin{cases} 0 & t \le -\frac{b}{2} \\ \frac{1}{a}\left(1 - e^{-at}e^{-\frac{ab}{2}}\right) & -\frac{b}{2} \le t \le \frac{b}{2} \\ y_{\max}e^{-at} & t \ge \frac{b}{2} \end{cases}$$