TOOL: The Rosetta Stone method of convolution is based on two key concepts.

1) The convolution of any function, $f(t)$, with an impulse function replicates $f(t)$ scaled by the area of the impulse.

2) The derivative of a convolution is equal the convolution of one of the functions with the derivative of the other function.

$$\frac{d}{dt}[f(t)*g(t)] = \frac{df(t)}{dt}*g(t) = f(t)*\frac{dg(t)}{dt}$$

Since simple functions that arise in signal processing, such as rectangles and triangles may be differentiated until impulses are obtained, we often can reduce the convolution to replication of some shapes, after which we integrate as many times as necessary to get back to the level of the original convolution.

Fig. 1 below shows an example of using differentiation and integration to find the convolution of two functions.

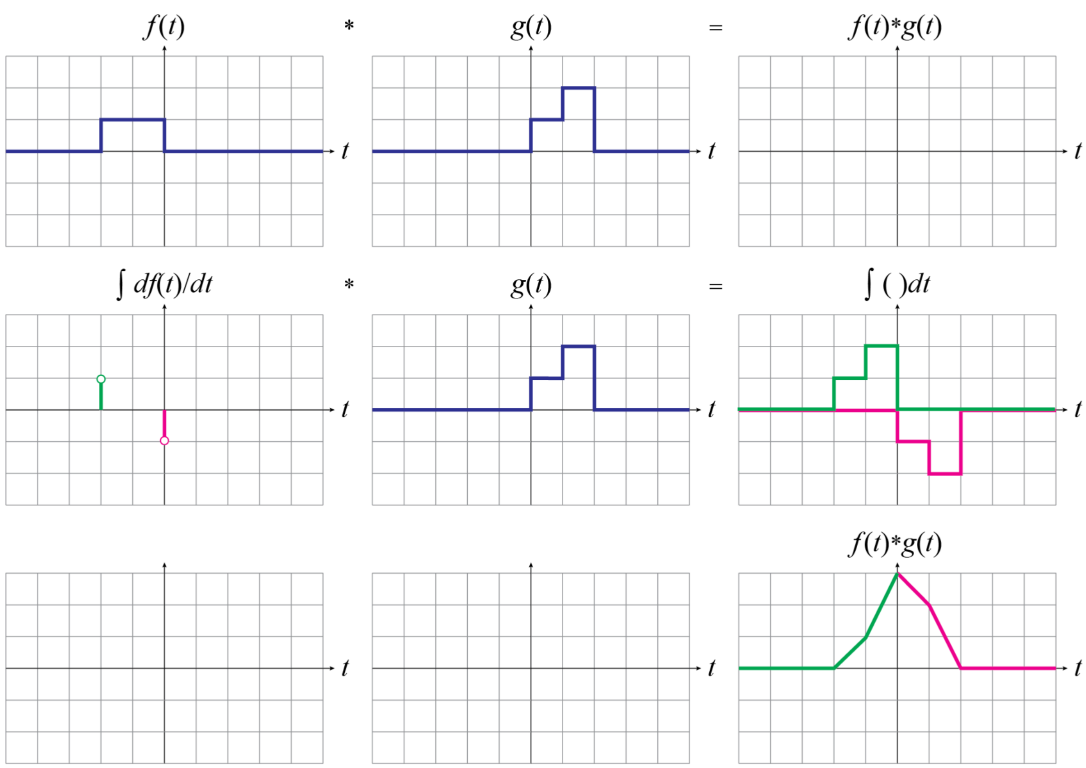

Fig. 1. Convolution by differentiation, convolution, integration.

Note the following aspects of the graphic:

1) The first line shows the functions being convolved but without a result.

2) In the second line, we differentiate only one of the two functions being convolved until impulses are obtained. Convolving with these impulses makes copies of the other function. We could also differentiate the second function to obtain only impulses, convolve all the impulses (resulting in more impulses), and integrate twice. However, one differentiation is sufficient.

3) Either of the functions in Fig. 1 may be differentiated, but differentiating the unit-height step function yields unit-area impulses, and the copies of the other rectangle function are then replicated without scaling.

4) The integration occurs from the second line to the third line. The integral starts at minus infinity and moves across the t-axis. The value of the integral (and the final convolution result) is the area under the curve left of t. The integral is zero until the green function is encountered at $t = -2$. At that point, the slope of the integral is the value of the green function (i.e., 1). At $t = -1$, the slope of the integral increases to the new value of the green function (i.e., 2). For $t > 0$, the red function dictates that the slope will be negative (-1 then -2).

5) Since $f(t)$ and $g(t)$ are of finite duration, the final convolution curve must end at a value of zero as t approaches infinity. Checking this condition is a good way to check for calculation errors.

6) To check the peak height of the convolution, we consider the formula for convolution without differentiation in which we reverse one function, slide it across the other function, and multiplying the function. If we reverse $f(t)$, we have the maximum overlap of reversed $f(t)$ and $g(t)$ when they are on top of each other. The product will be $g(t)$, and the area will be three. That area is the peak height of the convolution.