TOOL: Power

In analogy with power in a resistor being $p(t) = \dfrac{v^2(t)}{R}$ or $p(t) = i^2(t)R$, *instantaneous power for a function*, $x(t)$, at time t is the square of the magnitude of the function.

$$p(t) = |x(t)|^2$$

For a *sampled signal*, the power is the square of the sample value.

$$p[n] = |x[n]|^2$$

For a *sampled signal value treated as an impulse function* in a continuous-time framework, the *average* power is the square of the area of the impulse.

$$p(t) = |x[n]|^2 \text{ for } x(nT) = x[n]\delta(t - nT)$$

Energy

Energy in time $[t_1, t_2]$ is the integral of power over the interval.

$$w = \int_{t_1}^{t_2} p(t)dt$$

The *total energy* over all time is the integral of power over all time. Note that this value may be infinite, in which case average power may be used (if it is finite).

$$w_\infty = \int_{-\infty}^{\infty} p(t)dt$$

For a *sampled signal*, the energy in samples from n_1 to n_2 is the sum of the power in the samples.

$$w = \sum_{n=n_1}^{n_2} p[n]$$

For a *sampled signal*, the total energy is the sum of the power in the samples.

$$w_\infty = \sum_{n=\infty}^{\infty} p[n]$$

Average Power

The average of a function over an interval is the area under the function on the interval divided by the length of the interval.

$$x_{ave} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x(t)dt$$

The average of a function over all time, if it exists, is the limit of the area under the function divided the length of the interval of integration.

$$p_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)dt$$

For *signals with infinite energy*, we may be able to calculate an average power value.

$$p_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} p(t)dt$$

For a *sampled signal*, the average power is taken as a limit.

$$p_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{-N}^{N} p(t)dt$$

For a *sampled signal treated as impulse functions* in a continuous-time framework, the average power is the sum of the average powers from the impulses.

$$p_\infty = \sum_{n=\infty}^{\infty} p[n] = \sum_{n=\infty}^{\infty} |x[n]|^2 \text{ for } x(nT) = x[n]\delta(t - nT)$$

Periodic Waveform Power

If a function is periodic, we compute the energy in one period, T, and divide by the period.

$$p = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

Any interval equal to one period will suffice. For example, we also have

$$p = \frac{1}{T}\int_{0}^{T} |x(t)|^2 dt.$$

REF: Alan V. Oppenheim and Alan S. Willsky, with S. Hamid Nawab, *Signals and Systems,* 2nd Edition, Upper Saddle River, NJ: Prentice Hall, 1996. ISBN 0-13-814757-4