EX: Find the energy in the function shown in Fig. 1. Note that the function is zero outside the interval shown.

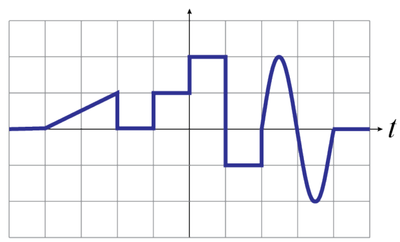

Fig. 1. Finite-energy signal.

SOLN: The scale of Fig. 1 is one unit per box for both axes. For a numerical solution, we first write a formula for the function. Since the function changes its shape as it goes, we have a piecewise definition.

$$x(t) = \begin{cases} 0 & t < -4 \\ \frac{1}{2}(t+4) & -4 < t < -2 \\ 1 & -1 < t < 0 \\ 2 & 0 < t < 1 \\ -1 & 1 < t < 2 \\ 2\sin(\pi t) & 2 < t < 4 \\ 0 & 4 < t \end{cases}$$

Note that the formula for the sloped line segment on the left is written as slope times x-axis intercept, i.e., $(t - -4)$.

For energy, we integrate the power, which is magnitude squared.

$$|x(t)|^2 = \begin{cases} 0 & t < -4 \\ \frac{1}{4}(t+4)^2 & -4 < t < -2 \\ 1 & -1 < t < 0 \\ 4 & 0 < t < 1 \\ 1 & 1 < t < 2 \\ 4\sin^2(\pi t) & 2 < t < 4 \\ 0 & 4 < t \end{cases}$$

If we write integrals for the energy calculations, we have a sum of terms, one for each nonzero interval.

$$w = \int_{-4}^{-2} (t+4)^2 dt + \int_{-1}^{0} 1dt + \int_{0}^{1} 4dt + \int_{1}^{2} 1dt + \int_{2}^{4} 4\sin^2(\pi t)dt$$

We only need the area under each shape in the plot of magnitude squared. We may shift the shapes right or left to create simpler integrals. This is equivalent to doing a change of variables, but it is easier to do the shift visually.

$$w = \int_{0}^{2} t^2 dt + \int_{0}^{1} 1dt + \int_{0}^{1} 4dt + \int_{0}^{1} 1dt + \int_{0}^{2} 4\sin^2(\pi t)dt$$

or

$$w = \frac{2^3}{3} + 1 + 4 + 1 + \int_{0}^{2} 4\sin^2(\pi t)dt$$

Rather than doing the squared sine integral, we observe that the average of $\sin^2(\,)$ and $\cos^2(\,)$ over an integer number of cycles is 1/2. (A plot of $\sin^2(\,)$ or $\cos^2(\,)$ makes it apparent that the result is a sinusoid with a vertical offset of 1/2. The variations around the average are symmetric and cancel out in the average value.) So we multiply the amplitude of 4 by the width of the integral, 2, and by 1/2.

We conclude that the last integral is $4 \cdot 2 \cdot \frac{1}{2}$. So we have

$$w = \frac{8}{3} + 1 + 4 + 1 + 4 = \frac{38}{3}.$$